

## **Common Limites and Errors**

Professor Ji-Woong Chung  
Korea University

# Outline

Data limitations

Hypothesis testing mistakes

How to control for unobserved heterogeneity

How not to control for it

## Data Limitations

- ▶ The data we use is almost never perfect:
  - ▶ Variables are often reported with error.
  - ▶ Exit and entry into dataset typically not random.
  - ▶ Datasets only cover certain types of firms.

## Measurement Error – Examples

- ▶ Measurement error occurs when observed values differ from the true values.
- ▶ Two main types:
  - ▶ **Random (innocent) errors:** Pure noise, not systematically related to other variables.
    - E.g. Survey respondents round or misremember income.
      - ▶ Leads to greater variance, but no systematic bias if uncorrelated with regressors.
  - ▶ **Systematic (nonrandom) errors:** Certain groups misreport in predictable ways.
    - E.g. High-GPA teenagers underreport marijuana use; firms understate liabilities.
      - ▶ Correlation with covariates  $\Rightarrow$  biased estimates.
- ▶ **Key question:** How do these errors affect causal inference and estimation?

# Measurement Error – Why It Matters

- ▶ The impact depends on which variable is measured with error.
- ▶ **If the dependent variable ( $y$ ) is mismeasured:**
  - ▶ Random noise: Increases residual variance  $\Rightarrow$  larger standard errors.
  - ▶ Systematic error: If correlated with  $x$ , coefficient estimates become biased.  
E.g. Low-education respondents underreport income  $\Rightarrow$  downward bias on education effect.
- ▶ **If the independent variable ( $x$ ) is mismeasured:**
  - ▶ Classical error (mean-zero, uncorrelated): Attenuation bias  $\Rightarrow$  slope biased toward 0.
  - ▶ Non-classical error (correlated): Bias in unpredictable directions; contaminates other coefficients.  
E.g. Noisy education measure  $\Rightarrow$  underestimated returns to schooling.
- ▶ **Summary:** Random  $\Rightarrow$  inefficiency; Systematic  $\Rightarrow$  bias.

# Measurement Error – Solutions

- ▶ Measurement error correction requires knowing its **source and structure**.
- ▶ **Common approaches:**
  - ▶ **Instrumental Variables (IV):** Find variable correlated with true  $x$  but not error.  
*E.g.* Administrative wage data as instrument for self-reported income.
  - ▶ **Validation samples or repeated measures:** Estimate or correct error variance.
  - ▶ **Structural modeling:** Explicitly model the measurement process
- ▶ **Challenges:**
  - ▶ Hard to correct without auxiliary or validation data.
  - ▶ Unknown error patterns  $\Rightarrow$  unpredictable bias.
- ▶ Always scrutinize data accuracy—small errors can distort inference.

## Survivorship Issues – Examples

- ▶ Observations may be missing or included for **systematic reasons**, not by chance.
- ▶ **Example 1:** IPO firms
  - ▶ Datasets of public firms exclude private firms.
  - ▶ Firms that go public may already differ (e.g., more profitable, faster-growing, or better governed).
- ▶ **Example 2:** Distressed or failed firms
  - ▶ Firms severely affected by a shock may disappear due to bankruptcy.
  - ▶ Remaining sample overrepresents “survivors.”
- ▶ **Question:** How do such missing or selective exits bias our estimates?

# Survivorship Issues – Why It Matters

- ▶ **Selection bias** can lead to misleading conclusions.
- ▶ **Example 1: IPOs and growth**
  - ▶ High post-IPO growth may not be caused by going public.
  - ▶ Rather, firms that went public were already high-growth candidates.
- ▶ **Example 2: Negative events and exits**
  - ▶ If failing firms disappear after a shock, the observed average effect looks smaller (or even positive).
  - ▶ Survivors are systematically different from those that dropped out.
- ▶ **Result:** Bias in estimates, especially in causal or panel analyses.

# Survivorship Issues – Solutions

- ▶ No perfect fix, but several **diagnostic checks** help:
- ▶ **1. Check for selective attrition:**
  - ▶ In DiD, test whether treatment status predicts dropping from the data.
  - ▶ If treatment increases exit probability, estimate may be biased.
- ▶ **2. Compare characteristics of dropouts vs. survivors:**
  - ▶ Are exiting observations systematically different in key covariates?
  - ▶ If yes, assess how their absence might affect estimates.
- ▶ **3. Sensitivity checks:**
  - ▶ Include censored or imputed outcomes where possible.
  - ▶ Use survival models (hazard or selection models) if dropout is endogenous.

## Sample is Limited – Examples

- ▶ Many widely used datasets cover only a **subset of firms**.
- ▶ **Example 1: Compustat**
  - ▶ Focuses on large, listed U.S. firms.
  - ▶ Excludes small, private, and young firms.
- ▶ **Example 2: Execucomp**
  - ▶ Covers CEO pay and incentives only for S&P 1500 firms.
  - ▶ Omits privately held or smaller listed firms.
- ▶ **Question:** How could this limited coverage bias our findings?

## Sample is Limited – Why It Matters

- ▶ Limited samples threaten **external validity**.
- ▶ **Example 1: Treatment effects in Compustat**
  - ▶ You may find no effect in large public firms.
  - ▶ But the same treatment could strongly affect unobserved small or private firms.
- ▶ **Example 2: CEO incentives in Execucomp**
  - ▶ Correlation between incentives and risk-taking may reflect large-firm governance structures.
  - ▶ May not generalize to smaller or family-controlled firms.
- ▶ Key issue: **Selection on observables and unobservables** into the dataset.

## Sample is Limited – Solutions

- ▶ **1. Be explicit about scope:**
  - ▶ Avoid overgeneralization; limit conclusions to the covered population.
  - ▶ Emphasize that results apply to large public firms if using Compustat or Execucomp.
- ▶ **2. Argue representativeness:**
  - ▶ Show your sample captures an economically important segment.
  - ▶ E.g., S&P 1500 firms represent majority of U.S. market capitalization.
- ▶ **3. Extend the data:**
  - ▶ Hand-collect missing data or merge with private firm databases.
  - ▶ Building new datasets can yield high-impact, publishable research.

## Example

- ▶ Ali, Klasa, and Yeung (RFS 2009) provide a striking case of data mismeasurement.
- ▶ Many finance theories emphasize **industry concentration** as a key variable:
  - ▶ E.g., competition, market power, R&D incentives, and financing constraints.
- ▶ Researchers typically measure concentration (Herfindahl index) using **Compustat**.
- ▶ **Question:** What's wrong with that approach?

## Example [Part 1]

- ▶ **Systematic measurement error:**
  - ▶ Compustat **excludes** private firms  $\Rightarrow$  distorted Herfindahl index.
  - ▶ Ali, Klasa, and Yeung construct an alternative using **U.S. Census data** (which includes all firms).
  - ▶ Correlation between the Compustat and Census-based measures = only **13%**.
- ▶ The error is not random:
  - ▶ Bias is related to observable industry traits—e.g., turnover, entry, exit, and listing propensity.

## Example [Part 2]

- ▶ Using the Census-based measure, Ali, Klasa, and Yeung (RFS 2009) show that:
  - ▶ The mismeasurement meaningfully changes empirical conclusions.
  - ▶ Four major published results are overturned.
- ▶ **Example:**
  - ▶ Previous studies (using Compustat) found a **negative** link between concentration and R&D.
  - ▶ With accurate Census data, the relationship becomes **positive**.
- ▶ **Lesson:** Measurement error in key variables can fundamentally alter conclusions.

# Outline

Data limitations

Hypothesis testing mistakes

How to control for unobserved heterogeneity

How not to control for it

## Hypothesis Testing Mistakes

- ▶ Researchers often compare treatment effects across groups by estimating separate DiDs.
- ▶ **Example:**
  - ▶ Estimate treatment effect for small firms.
  - ▶ Estimate treatment effect for large firms.
- ▶ Then they conclude: "The effect is stronger for large firms."

## Example Inference from Analysis

	Small Firms	Large Firms	Low D/E Firms	High D/E Firms
Treatment $\times$ Post	0.031 (0.121)	0.104** (0.051)	0.056 (0.045)	0.081*** (0.032)
$N$	2,334	3,098	2,989	2,876
$R^2$	0.11	0.15	0.08	0.21
Year FE	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓

- ▶ Researchers often conclude:
  - ▶ “Treatment effect is larger for big firms.”
  - ▶ “High D/E firms respond more.”
  - ▶ *But are those differences statistically significant?*

## Be Careful Making Such Claims

- ▶ **Problem:** Differences across subsamples may not be statistically significant.
- ▶ You can't tell by "eyeballing" coefficients.
  - ▶ Statistical significance depends on the **covariance** between estimates.
- ▶ **Proper test:** Include an interaction term (triple difference) in a single regression.

## Example Triple Interaction Result

	All Firms
Treatment $\times$ Post	0.031 (0.121)
Treatment $\times$ Post $\times$ Large	0.073 (0.065)
$N$	5,432
$R^2$	0.12
Year FE	✓
Firm FE	✓
Year $\times$ Large FE	✓

- ▶ Difference between large and small firms is **not statistically significant**.
- ▶ Always include interaction with year dummies to match subgroup DiDs.

## Practical Advice

- ▶ Don't make claims you haven't statistically tested.
- ▶ Always report the **p-value for the difference** across groups.
- ▶ If the difference isn't significant (e.g.,  $p = 0.15$ ), say so — triple differences are noisy.
- ▶ Be cautious with phrasing:
  - ▶ Instead of: "Large firms respond more,"
  - ▶ Say: "We find an effect for large firms but not for small firms."

# Outline

Data limitations

Hypothesis testing mistakes

How to control for unobserved heterogeneity

How not to control for it

# Outline

Data limitations

Hypothesis testing mistakes

How to control for unobserved heterogeneity

How not to control for it

## Unobserved Heterogeneity – Motivation

- ▶ Controlling for **unobserved heterogeneity** is a fundamental challenge in empirical finance.
- ▶ **Why?** Many important factors cannot be directly measured or included in data:
  - ▶ Managerial talent, corporate culture, or risk appetite.
  - ▶ Local demand or regulatory conditions.
  - ▶ Investor sentiment or regional economic trends.
- ▶ These unobservables can be **correlated** with key explanatory variables:
  - ▶ ⇒ Leads to **omitted variable bias**.
- ▶ Important sources of heterogeneity are often shared across **groups**:
  - ▶ Industry-level demand shocks.
  - ▶ Region-specific economic or policy environments.
  - ▶ Time-period shocks common to all firms.

## Many Different Strategies Are Used

- ▶ As discussed earlier, **Fixed Effects (FE)** can control for unobserved heterogeneity and yield consistent estimates when the unobservables are time-invariant.
- ▶ But researchers use several alternative or complementary strategies to remove **group-level heterogeneity**:
  - ▶ **Adjusted-Y (AdjY)**: Demean the dependent variable within groups (e.g., subtract the industry-year average: “industry-adjusted” outcomes).
  - ▶ **Average Effects (AvgE)**: Include group-level averages of outcomes as controls (e.g., add the mean of  $y$  for a given state-year or industry-year).
- ▶ Each method aims to remove variation driven by shared shocks or persistent group differences.
  - ▶ FE fully removes group-level heterogeneity (e.g., via industry-year dummies).
  - ▶ AdjY and AvgE are simplified approximations useful in small samples, but only FE yields consistent estimates when unobservables correlate with regressors.

## The Underlying Model [Part 1]

- ▶ Start with a simple data-generating process:

$$y_{i,j} = \beta X_{i,j} + f_i + \epsilon_{i,j}$$

- ▶  $i$ : Group index (e.g., industry, state, bank, or fund family)
- ▶  $j$ : Observation within group (e.g., firm, branch, fund)
- ▶ Model components:
  - ▶  $y_{i,j}$ : outcome (e.g., investment, leverage, return)
  - ▶  $X_{i,j}$ : explanatory variable of interest (e.g., policy, treatment)
  - ▶  $f_i$ : unobserved group-level factor (e.g., industry demand, regulation)
  - ▶  $\epsilon_{i,j}$ : idiosyncratic error term
- ▶ The key question: what happens if we try to control for  $f_i$  without properly including a fixed effect?

## The Underlying Model [Part 2]

- ▶ Standard assumptions about the data structure:
  - ▶  $N$ : Number of groups is large;  $J$ : Observations per group is small.
  - ▶  $\text{Var}(f_i) = \sigma_f^2$ ,  $\mathbb{E}[f_i] = 0$
  - ▶  $\text{Var}(X_{i,j}) = \sigma_X^2$ ,  $\mathbb{E}[X_{i,j}] = 0$
  - ▶  $\text{Var}(\epsilon_{i,j}) = \sigma_\epsilon^2$ ,  $\mathbb{E}[\epsilon_{i,j}] = 0$
- ▶  $X$  and  $\epsilon$  are i.i.d. across groups, but may be correlated **within** groups:
  - ▶ Within-group correlation  $\Rightarrow$  common shocks.
  - ▶ Across-group independence ensures valid asymptotics.

## The Underlying Model [Part 3]

- ▶ Additional assumptions:
  - ▶  $\text{Cov}(f_i, \epsilon_{i,j}) = 0$  — group factors are uncorrelated with idiosyncratic errors.
  - ▶  $\text{Cov}(X_{i,j}, \epsilon_{i,j}) = \text{Cov}(X_{i,j}, \epsilon_{i,-j}) = 0$  — exogeneity of  $X$ .
    - ▶  $X_{i,j}$  is exogenous with respect to both its own error term and the error terms of other group members, enabling unbiased and consistent estimation of  $\beta$  in the fixed effects model.
  - ▶  $\text{Cov}(X_{i,j}, f_i) = \sigma_{Xf} \neq 0$  — regressor correlated with group unobservables.
- ▶ Implication:
  - ▶ If we omit  $f_i$ , OLS suffers from classic **omitted variable bias**.
  - ▶ FE removes  $f_i$  through within-group demeaning. AdjY and AvgE only partially do so, leaving residual correlation with  $f_i$ . This incomplete adjustment can amplify—or even reverse—the bias.

## We Already Know OLS Is Biased

True model:  $y_{i,j} = \beta X_{i,j} + f_i + \epsilon_{i,j}$

But OLS estimates:  $y_{i,j} = \hat{\beta}_{OLS} X_{i,j} + u_{i,j}^{OLS}$

- ▶ By omitting the group effect  $f_i$ , OLS suffers from standard omitted variable bias:

$$\hat{\beta}_{OLS} = \beta + \frac{\sigma_{Xf}}{\sigma_X^2}$$

- ▶ Direction and size of bias depend on the covariance between  $X$  and  $f_i$ .

## Adjusted-Y ( $AdjY$ ) Estimation

- **Idea:** Remove unobserved group effects by demeaning the dependent variable within groups:

$$y_{i,j} - \bar{y}_i = \beta^{AdjY} X_{i,j} + u_{i,j}^{AdjY}$$

- Group mean:

$$\bar{y}_i = \frac{1}{J} \sum_{k \in i} y_{i,k} = \frac{1}{J} \sum_{k \in i} (\beta X_{i,k} + f_i + \epsilon_{i,k})$$

$$\Rightarrow \bar{y}_i = \beta \bar{X}_i + f_i + \bar{\epsilon}_i$$

- Some researchers exclude the observation itself or use medians, but bias remains.

## Example: $AdjY$ Estimation in Practice

- ▶ Example regression:

$$Q_{i,j,t} - \bar{Q}_{i,t} = \alpha + \beta X_{i,j,t} + \epsilon_{i,j,t}$$

- ▶ Variables:

- ▶  $Q_{i,j,t}$ : Tobin's Q for firm  $j$  in industry  $i$ , year  $t$
- ▶  $\bar{Q}_{i,t}$ : industry-year mean Q ("industry-adjusted Q")
- ▶  $X_{i,j,t}$ : explanatory variables (e.g., governance, leverage)
- ▶ Often combined with firm or year fixed effects

- ▶ **Question:** Why is  $AdjY$  still inconsistent?

## Why $AdjY$ Is Inconsistent

- ▶ Substitute the group mean:

$$\bar{y}_i = \beta \bar{X}_i + f_i + \bar{\epsilon}_i$$

$$y_{i,j} - \bar{y}_i = (\beta X_{i,j} + f_i + \epsilon_{i,j}) - (\beta \bar{X}_i + f_i + \bar{\epsilon}_i)$$

- ▶ The transformation removes  $f_i$  in the mean but not in the regressor.
- ▶ When we regress  $(y_{i,j} - \bar{y}_i)$  on  $X_{i,j}$  instead of  $(X_{i,j} - \bar{X}_i)$ , the omitted  $\bar{X}_i$  induces bias.
- ▶ Hence,  $AdjY$  omits a relevant group-level term.

## AdjY and Omitted Variable Bias

- ▶ The true transformed model is:

$$y_{i,j} - \bar{y}_i = \beta(X_{i,j} - \bar{X}_i) + (\epsilon_{i,j} - \bar{\epsilon}_i)$$

- ▶ But *AdjY* estimates:

$$y_{i,j} - \bar{y}_i = \beta^{AdjY} X_{i,j} + u_{i,j}^{AdjY}$$

- ▶ Because it omits  $\bar{X}_i$ , the estimator is biased:

$$\hat{\beta}_{AdjY} = \beta - \frac{\sigma_{X\bar{X}}^2}{\sigma_X^2}$$

- ▶ With positive  $\text{Cov}(X, \bar{X})$ —common under shared industry shocks—bias is typically negative.

## Adding a Second Variable, $Z$

- ▶ Suppose the true model has two regressors:

$$y_{i,j} = \beta X_{i,j} + \gamma Z_{i,j} + f_i + \epsilon_{i,j}$$

- ▶ Maintain previous assumptions and add:
  - ▶  $\text{Cov}(Z_{i,j}, \epsilon_{i,j}) = 0, \text{Var}(Z) = \sigma_Z^2$
  - ▶  $\text{Cov}(X, Z) = \sigma_{XZ}$
  - ▶  $\text{Cov}(Z, f_i) = \sigma_{Zf}$
- ▶ AdjY still omits group-level means  $(\bar{X}_i, \bar{Z}_i)$ , creating intertwined biases.

## AdjY Estimates with Two Variables

- ▶ The biases are now complex:

$$\hat{\beta}_{AdjY} = \beta + \Delta, \quad \hat{\gamma}_{AdjY} = \gamma + \diamond$$

- ▶ Biases depend on correlations among  $X, Z, f_i$ .
- ▶ As Gormley and Matsa (2014) show:
  - ▶ Both coefficients can move in unpredictable directions.
  - ▶ Even sign reversals are possible.

## Average Effects ( $\text{Avg}E$ ) — Idea

- ▶ Researchers often want to control for unobserved group-level factors ( $f_i$ ) when fixed effects are unavailable or costly.
- ▶ **Idea:** Include the group mean of the dependent variable as a proxy for  $f_i$ :

$$y_{i,j} = \beta^{\text{Avg}E} X_{i,j} + \gamma^{\text{Avg}E} \bar{y}_i + u_{i,j}^{\text{Avg}E}$$

- ▶ Example – Firm profitability regression:

$$\text{ROA}_{i,s,t} = \alpha + \beta X_{i,s,t} + \gamma \overline{\text{ROA}}_{s,t} + u_{i,s,t}$$

- ▶  $\overline{\text{ROA}}_{s,t}$ : Average ROA among firms in state  $s$ , year  $t$
- ▶  $X_{i,s,t}$ : Firm-level controls (e.g., leverage, size, market share)
- ▶ **Goal:** Use  $\bar{y}_i$  to soak up unobserved shocks ( $f_i$ ) that affect all group members.

## Why $\text{Avg}E$ Is Problematic

- ▶ The true model:

$$y_{i,j} = \beta X_{i,j} + f_i + \epsilon_{i,j}$$

- ▶  $\text{Avg}E$  substitutes a proxy for  $f_i$ :

$$\bar{y}_i = \beta \bar{X}_i + f_i + \bar{\epsilon}_i$$

- ▶ Substituting this into the regression gives:

$$y_{i,j} = \beta X_{i,j} + \gamma(\beta \bar{X}_i + f_i + \bar{\epsilon}_i) + u_{i,j}$$

- ▶ Two problems arise:

1. **Measurement error:**  $\bar{y}_i$  is an imperfect proxy for  $f_i$  — it includes  $\beta \bar{X}_i + \bar{\epsilon}_i$ .
2. **Endogeneity:** controlling for  $\bar{y}_i$  removes only the fraction of  $f_i$ ; The leftover  $f_i$  in the error is still correlated with  $X_{i,j}$ .

## Measurement Error Bias

- ▶ Since

$$\bar{y}_i = f_i + \underbrace{(\beta \bar{X}_i + \bar{\epsilon}_i)}_{\text{measurement error}} ,$$

$\bar{y}_i$  measures  $f_i$  with noise.

- ▶ This creates **measurement error bias**:
  - ▶ As is well known, even classical measurement error causes all estimated coefficients to be inconsistent
- ▶ Bias here is complicated because error can be correlated with both mismeasured variable,  $f_i$ , and with  $X_{i,j}$ .

## Summary of OLS, $AdjY$ , and $AvgE$

True model:  $y_{i,j} = \beta X_{i,j} + f_i + \epsilon_{i,j}$

True model:  $y_{i,j} - \bar{y}_i = \beta(X_{i,j} - \bar{X}_i) + \epsilon_{i,j} - \bar{\epsilon}_i$

$AdjY$  estimates:  $y_{i,j} - \bar{y}_i = \beta^{AdjY} X_{i,j} + u_{i,j}^{AdjY}$

$AvgE$  estimates:  $y_{i,j} = \beta^{AvgE} X_{i,j} + \gamma^{AvgE} \bar{y}_i + u_{i,j}^{AvgE}$

- ▶ All three estimators are inconsistent in the presence of unobserved group heterogeneity.
- ▶  $AdjY$  and  $AvgE$  are not necessarily an improvement over OLS.
- ▶  $AdjY$  and  $AvgE$  can yield estimates with opposite signs to the true coefficient.

## The Differences Will Matter! Example 1 — Capital Structure

- ▶ Regression model:

$$(D/A)_{i,t} = \alpha + \beta X_{i,t} + f_i + \epsilon_{i,t}$$

- ▶  $(D/A)_{i,t}$ : Book leverage for firm  $i$ , year  $t$
- ▶  $X_{i,t}$ : Variables affecting leverage (e.g., tangibility, size, profitability)
- ▶  $f_i$ : Firm fixed effect capturing unobserved, time-invariant factors
- ▶ Data: U.S. firms, 1950–2010, winsorized at 1% tails
- ▶ Goal: Compare how different estimators handle unobserved heterogeneity ( $f_i$ )

# Capital Structure Regression Results

Dependent Variable: Book Leverage

Variable	OLS	AdjY	AvgE	FE
Fixed Assets / Total Assets	0.270*** (0.008)	0.066*** (0.004)	0.103*** (0.004)	0.248*** (0.014)
Ln(Sales)	0.011*** (0.001)	0.011*** (0.000)	0.011*** (0.000)	0.017*** (0.001)
Return on Assets	-0.015*** (0.005)	0.051*** (0.004)	0.039*** (0.004)	-0.028*** (0.005)
Z-score	-0.017*** (0.000)	-0.010*** (0.000)	-0.011*** (0.000)	-0.017*** (0.001)
Market-to-Book Ratio	-0.006*** (0.000)	-0.004*** (0.000)	-0.004*** (0.000)	-0.003*** (0.000)
Observations	166,974	166,974	166,974	166,974
R <sup>2</sup>	0.29	0.14	0.56	0.66

- ▶ Notice how *AdjY* and *AvgE* estimates differ sharply from both OLS and FE.
- ▶ For example, the profitability (ROA) coefficient flips sign under *AdjY/AvgE*.
- ▶ Partial corrections for heterogeneity distort inference — bias can even reverse direction.

## The Differences Will Matter! Example 2 — Firm Value

- ▶ Regression model:

$$Q_{i,j,t} = \alpha + \beta X_{i,j,t} + f_{j,t} + \epsilon_{i,j,t}$$

- ▶  $Q_{i,j,t}$ : Tobin's Q for firm  $i$ , industry  $j$ , year  $t$
- ▶  $X_{i,j,t}$ : Firm-level determinants of value (e.g., size, R&D, profitability)
- ▶  $f_{j,t}$ : Industry-year fixed effect (controls for sectoral conditions)
- ▶ Data: U.S. manufacturing firms
- ▶ Question: Do OLS, AdjY, AvgE, and FE produce consistent results?

# Firm Value Regression Results

Dependent Variable: Tobin's Q

Variable	OLS	AdjY	AvgE	FE
Delaware Incorporation	0.100*** (0.036)	0.019 (0.032)	0.040 (0.032)	0.086** (0.039)
Ln(Sales)	-0.125*** (0.009)	-0.054*** (0.008)	-0.072*** (0.008)	-0.131*** (0.011)
R&D Expenses / Assets	6.724*** (0.260)	3.022*** (0.242)	3.968*** (0.256)	5.541*** (0.318)
Return on Assets	-0.559*** (0.108)	-0.526*** (0.095)	-0.535*** (0.097)	-0.436*** (0.117)
Observations	55,792	55,792	55,792	55,792
$R^2$	0.22	0.08	0.34	0.37

- ▶  $AdjY$  and  $AvgE$  substantially underestimate the R&D coefficient compared to FE (3.0 vs 5.5).
- ▶ Their partial corrections remove part of the true within-group variation.
- ▶ Overall fit ( $R^2$ ) confirms this — FE explains far more variation, capturing persistent unobserved factors.

## General Implications of the Framework

- ▶ The same logic applies well beyond firm-level panel regressions:
  - ▶ Many “adjusted” estimators implicitly assume the group mean or median removes unobserved heterogeneity.
- ▶ However, any estimator that subtracts off a noisy or endogenous benchmark still suffers from omitted-variable or measurement-error bias.
- ▶ **Examples of biased AdjY-type estimators:**
  - ▶ Subtracting the group **median** or **value-weighted mean** instead of the unobserved fixed effect.
  - ▶ Subtracting the mean outcome of a **matched control sample** (as in diversification-discount studies).
  - ▶ Comparing “adjusted” outcomes before vs. after an event (as in M&A announcement studies).
  - ▶ Using **characteristically adjusted returns** in asset pricing.
- ▶ These adjustments remove some noise but not the unobserved heterogeneity that actually drives bias.

## AdjY-Type Estimators in Asset Pricing

- ▶ In empirical asset pricing, researchers often compare returns across portfolios sorted by firm characteristics.
- ▶ Returns are typically “**characteristically adjusted**”:
  - ▶ Subtract the mean return of a benchmark portfolio with similar size, book-to-market, or R&D intensity.
  - ▶  $r_{i,t} - \bar{r}_{\text{benchmark},t}$  is regressed on firm characteristics.
- ▶ **Problem:** This is mathematically equivalent to *AdjY*.
  - ▶ The benchmark mean ( $\bar{r}_{\text{benchmark},t}$ ) is a noisy, endogenous proxy for the unobserved common component (e.g., systematic factor, industry effect).
  - ▶ It does not hold constant the variation in the independent variable across benchmark portfolios.
- ▶ Hence, the adjustment does not eliminate unobserved co-movement — it may even exaggerate it.

## Asset Pricing *AdjY* Example — R&D and Stock Returns

- ▶ Example: Firms sorted into quintiles by R&D intensity (R&D/MVE).
- ▶ Researchers compute “characteristically adjusted” yearly returns by subtracting industry-size benchmark means (i.e., an *AdjY* transformation).

Missing	Q1	Q2	Q3	Q4	Q5
-0.012*** (0.003)	-0.033*** (0.009)	-0.023*** (0.008)	-0.002 (0.007)	0.008 (0.013)	0.020*** (0.006)

- ▶ Benchmark portfolios: industry-size matched means of returns.
- ▶ Difference between Q5 and Q1 = 5.3 percentage points.
- ▶ Appears to suggest “high R&D firms outperform.”
- ▶ But since benchmark returns correlate with firm characteristics and unobserved shocks, this inference may be spurious.

## Regression Comparison: *AdjY* vs Fixed Effects

Dependent Variable: Yearly Stock Return

R&D Quintile	<i>AdjY</i> Estimate	FE Estimate
Missing	0.021** (0.009)	0.030*** (0.010)
Quintile 2	0.010 (0.013)	0.019 (0.019)
Quintile 3	0.032*** (0.012)	0.051*** (0.014)
Quintile 4	0.041*** (0.015)	0.068*** (0.018)
Quintile 5	0.053*** (0.011)	0.094*** (0.020)
Observations	144,592	144,592
$R^2$	0.00	0.40

- ▶ Regression equivalent of the previous “sorts” result.
- ▶ The FE version applies benchmark-period fixed effects to both returns and R&D — conceptually a cleaner “within” estimator.
- ▶ *AdjY* coefficients are consistently smaller in magnitude.
- ▶  $R^2$  is near zero under *AdjY* but large under FE — indicating that benchmark adjustment misses systematic variation.

## What If $AdjY$ or $AvgE$ Is the True Model?

- ▶ Suppose the data truly follow the  $AvgE$  structure—where the **group mean outcome** directly affects each member's outcome:

$$y_{i,j} = \beta X_{i,j} + \gamma \bar{y}_i + u_{i,j}$$

- ▶ Then  $\bar{y}_i$  itself depends on  $\bar{u}_i$ , which includes  $u_{i,j}$ .
- ▶ This creates a simultaneous relationship: individuals affect the group mean and the group mean affects individuals.
- ▶ This is the classic **reflection problem** (Manski, 1993) — identifying peer or group effects becomes impossible without extra structure or instruments.
- ▶ In this case, **none** of the estimators (OLS,  $AdjY$ ,  $AvgE$ , or FE) recover the true  $\beta$ . [See Leary and Roberts (2010) for a finance application.]

## What If $AdjY$ or $AvgE$ Is the True Model?

- ▶ Even if the researcher is interested in deviations from the group mean:  $(y_{i,j} - \bar{y}_i)$ , the  $AdjY$  estimator is only consistent if  $X_{i,j}$  has no effect on  $y_{i,j}$ .
  - ▶ If  $X_{i,j}$  influences  $y_{i,j}$ , then it must also influence others in the same group  $(y_{i,-j})$  through correlated behavior or shared shocks.
  - ▶ Therefore,  $\bar{X}_i$  also affects  $(y_{i,j} - \bar{y}_i)$ , implying:

$$\text{Cov}(X_{i,j}, (y_{i,j} - \bar{y}_i)) \neq 0.$$

- ▶ In short, it is impossible for  $X_{i,j}$  to affect  $y_{i,j}$  but not the group deviation  $(y_{i,j} - \bar{y}_i)$ .
- ▶ When true interdependence exists among group members, simple “adjusted” or “demeaned” models confound cause and reflection. Identifying the effect of  $X_{i,j}$  requires either instrumental variables or structural modeling of peer interactions.

## Summary of Today [Part 1]

- ▶ Our data isn't perfect:
  - ▶ Watch for measurement error.
  - ▶ Watch for survivorship bias.
  - ▶ Be careful about external validity claims.
- ▶ Test that estimates across subsamples are statistically different (if you plan to claim differences).

## Summary of Today [Part 2]

- ▶ Don't use  $AdjY$  or  $AvgE$ !
- ▶ Do use fixed effects:
  - ▶ Use benchmark portfolio-period FE in asset pricing rather than characteristically adjusted returns.