

Common Limites and Errors

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Outline

Data limitations

Hypothesis testing mistakes

How to control for unobserved heterogeneity

How not to control for it

Data Limitations

- ▶ The data we use is almost never perfect:
 - ▶ Variables are often reported with error.
 - ▶ Exit and entry into dataset typically not random.
 - ▶ Datasets only cover certain types of firms.

Measurement Error – Examples

- ▶ Measurement error occurs when observed values differ from the true values.
- ▶ Two main types:
 - ▶ **Random (innocent) errors:** Pure noise, not systematically related to other variables.
 - E.g. Survey respondents round or misremember income.
 - ▶ Leads to greater variance, but no systematic bias if uncorrelated with regressors.
 - ▶ **Systematic (nonrandom) errors:** Certain groups misreport in predictable ways.
 - E.g. High-GPA teenagers underreport marijuana use; firms understate liabilities.
 - ▶ Correlation with covariates \Rightarrow biased estimates.
- ▶ **Key question:** How do these errors affect causal inference and estimation?

Measurement Error – Why It Matters

- ▶ The impact depends on which variable is measured with error.
- ▶ **If the dependent variable (y) is mismeasured:**
 - ▶ Random noise: Increases residual variance \Rightarrow larger standard errors.
 - ▶ Systematic error: If correlated with x , coefficient estimates become biased.

E.g. Low-education respondents underreport income \Rightarrow downward bias on education effect.
- ▶ **If the independent variable (x) is mismeasured:**
 - ▶ Classical error (mean-zero, uncorrelated): Attenuation bias \Rightarrow slope biased toward 0.
 - ▶ Non-classical error (correlated): Bias in unpredictable directions; contaminates other coefficients.

E.g. Noisy education measure \Rightarrow underestimated returns to schooling.
- ▶ **Summary:** Random \Rightarrow inefficiency; Systematic \Rightarrow bias.

Measurement Error – Solutions

- ▶ Measurement error correction requires knowing its **source and structure**.
- ▶ **Common approaches:**
 - ▶ **Instrumental Variables (IV):** Find variable correlated with true x but not error.
E.g. Administrative wage data as instrument for self-reported income.
 - ▶ **Validation samples or repeated measures:** Estimate or correct error variance.
 - ▶ **Structural modeling:** Explicitly model the measurement process
- ▶ **Challenges:**
 - ▶ Hard to correct without auxiliary or validation data.
 - ▶ Unknown error patterns \Rightarrow unpredictable bias.
- ▶ Always scrutinize data accuracy—small errors can distort inference.

Survivorship Issues – Examples

- ▶ Observations may be missing or included for **systematic reasons**, not by chance.
- ▶ **Example 1: IPO firms**
 - ▶ Datasets of public firms exclude private firms.
 - ▶ Firms that go public may already differ (e.g., more profitable, faster-growing, or better governed).
- ▶ **Example 2: Distressed or failed firms**
 - ▶ Firms severely affected by a shock may disappear due to bankruptcy.
 - ▶ Remaining sample overrepresents “survivors.”
- ▶ **Question:** How do such missing or selective exits bias our estimates?

Survivorship Issues – Why It Matters

- ▶ **Selection bias** can lead to misleading conclusions.
- ▶ **Example 1: IPOs and growth**
 - ▶ High post-IPO growth may not be caused by going public.
 - ▶ Rather, firms that went public were already high-growth candidates.
- ▶ **Example 2: Negative events and exits**
 - ▶ If failing firms disappear after a shock, the observed average effect looks smaller (or even positive).
 - ▶ Survivors are systematically different from those that dropped out.
- ▶ **Result:** Bias in estimates, especially in causal or panel analyses.

Survivorship Issues – Solutions

- ▶ No perfect fix, but several **diagnostic checks** help:
- ▶ **1. Check for selective attrition:**
 - ▶ In DiD, test whether treatment status predicts dropping from the data.
 - ▶ If treatment increases exit probability, estimate may be biased.
- ▶ **2. Compare characteristics of dropouts vs. survivors:**
 - ▶ Are exiting observations systematically different in key covariates?
 - ▶ If yes, assess how their absence might affect estimates.
- ▶ **3. Sensitivity checks:**
 - ▶ Include censored or imputed outcomes where possible.
 - ▶ Use survival models (hazard or selection models) if dropout is endogenous.

Sample is Limited – Examples

- ▶ Many widely used datasets cover only a **subset of firms**.
- ▶ **Example 1: Compustat**
 - ▶ Focuses on large, listed U.S. firms.
 - ▶ Excludes small, private, and young firms.
- ▶ **Example 2: Execucomp**
 - ▶ Covers CEO pay and incentives only for S&P 1500 firms.
 - ▶ Omits privately held or smaller listed firms.
- ▶ **Question:** How could this limited coverage bias our findings?

Sample is Limited – Why It Matters

- ▶ Limited samples threaten **external validity**.
- ▶ **Example 1: Treatment effects in Compustat**
 - ▶ You may find no effect in large public firms.
 - ▶ But the same treatment could strongly affect unobserved small or private firms.
- ▶ **Example 2: CEO incentives in Execucomp**
 - ▶ Correlation between incentives and risk-taking may reflect large-firm governance structures.
 - ▶ May not generalize to smaller or family-controlled firms.
- ▶ Key issue: **Selection on observables and unobservables** into the dataset.

Sample is Limited – Solutions

▶ 1. Be explicit about scope:

- ▶ Avoid overgeneralization; limit conclusions to the covered population.
- ▶ Emphasize that results apply to large public firms if using Compustat or Execucomp.

▶ 2. Argue representativeness:

- ▶ Show your sample captures an economically important segment.
- ▶ E.g., S&P 1500 firms represent majority of U.S. market capitalization.

▶ 3. Extend the data:

- ▶ Hand-collect missing data or merge with private firm databases.
- ▶ Building new datasets can yield high-impact, publishable research.

Example

- ▶ Ali, Klasa, and Yeung (RFS 2009) provide a striking case of data mismeasurement.
- ▶ Many finance theories emphasize **industry concentration** as a key variable:
 - ▶ E.g., competition, market power, R&D incentives, and financing constraints.
- ▶ Researchers typically measure concentration (Herfindahl index) using **Compustat**.
- ▶ **Question:** What's wrong with that approach?

Example [Part 1]

- ▶ **Systematic measurement error:**

- ▶ Compustat **excludes** private firms \Rightarrow distorted Herfindahl index.
- ▶ Ali, Klasa, and Yeung construct an alternative using **U.S. Census data** (which includes all firms).
- ▶ Correlation between the Compustat and Census-based measures = only **13%**.

- ▶ The error is not random:

- ▶ Bias is related to observable industry traits—e.g., turnover, entry, exit, and listing propensity.

Example [Part 2]

- ▶ Using the Census-based measure, Ali, Klasa, and Yeung (RFS 2009) show that:
 - ▶ The mismeasurement meaningfully changes empirical conclusions.
 - ▶ Four major published results are overturned.
- ▶ **Example:**
 - ▶ Previous studies (using Compustat) found a **negative** link between concentration and R&D.
 - ▶ With accurate Census data, the relationship becomes **positive**.
- ▶ **Lesson:** Measurement error in key variables can fundamentally alter conclusions.

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Hypothesis Testing Mistakes

- ▶ Researchers often compare treatment effects across groups by estimating separate DiDs.
- ▶ **Example:**
 - ▶ Estimate treatment effect for small firms.
 - ▶ Estimate treatment effect for large firms.
- ▶ Then they conclude: “The effect is stronger for large firms.”

Example Inference from Analysis

	Small Firms	Large Firms	Low D/E Firms	High D/E Firms
Treatment \times Post	0.031 (0.121)	0.104** (0.051)	0.056 (0.045)	0.081*** (0.032)
N	2,334	3,098	2,989	2,876
R^2	0.11	0.15	0.08	0.21
Year FE	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓

- ▶ Researchers often conclude:
 - ▶ “Treatment effect is larger for big firms.”
 - ▶ “High D/E firms respond more.”
 - ▶ *But are those differences statistically significant?*

Be Careful Making Such Claims

- ▶ **Problem:** Differences across subsamples may not be statistically significant.
- ▶ You can't tell by “eyeballing” coefficients.
 - ▶ Statistical significance depends on the **covariance** between estimates.
- ▶ **Proper test:** Include an interaction term (triple difference) in a single regression.

Example Triple Interaction Result

	All Firms
Treatment \times Post	0.031 (0.121)
Treatment \times Post \times Large	0.073 (0.065)
N	5,432
R^2	0.12
Year FE	✓
Firm FE	✓
Year \times Large FE	✓

- ▶ Difference between large and small firms is **not statistically significant**.
- ▶ Always include interaction with year dummies to match subgroup DiDs.

Practical Advice

- ▶ Don't make claims you haven't statistically tested.
- ▶ Always report the **p-value for the difference** across groups.
- ▶ If the difference isn't significant (e.g., $p = 0.15$), say so — triple differences are noisy.
- ▶ Be cautious with phrasing:
 - ▶ Instead of: “Large firms respond more,”
 - ▶ Say: “We find an effect for large firms but not for small firms.”

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Unobserved Heterogeneity – Motivation

- ▶ Controlling for **unobserved heterogeneity** is a fundamental challenge in empirical finance.
- ▶ **Why?** Many important factors cannot be directly measured or included in data:
 - ▶ Managerial talent, corporate culture, or risk appetite.
 - ▶ Local demand or regulatory conditions.
 - ▶ Investor sentiment or regional economic trends.
- ▶ These unobservables can be **correlated** with key explanatory variables:
 - ▶ ⇒ Leads to **omitted variable bias**.
- ▶ Important sources of heterogeneity are often shared across **groups**:
 - ▶ Industry-level demand shocks.
 - ▶ Region-specific economic or policy environments.
 - ▶ Time-period shocks common to all firms.

Many Different Strategies Are Used

- ▶ As discussed earlier, **Fixed Effects (FE)** can control for unobserved heterogeneity and yield consistent estimates when the unobservables are time-invariant.
- ▶ But researchers use several alternative or complementary strategies to remove **group-level heterogeneity**:
 - ▶ **Adjusted-Y (AdjY)**: Demean the dependent variable within groups (e.g., subtract the industry-year average: “industry-adjusted” outcomes).
 - ▶ **Average Effects (AvgE)**: Include group-level averages of outcomes as controls (e.g., add the mean of y for a given state-year or industry-year).
- ▶ Each method aims to remove variation driven by shared shocks or persistent group differences.
 - ▶ FE fully removes group-level heterogeneity (e.g., via industry-year dummies).
 - ▶ AdjY and AvgE are simplified approximations useful in small samples, but only FE yields consistent estimates when unobservables correlate with regressors.

The Underlying Model [Part 1]

- ▶ Start with a simple data-generating process:

$$y_{i,j} = \beta X_{i,j} + f_i + \epsilon_{i,j}$$

- ▶ i : Group index (e.g., industry, state, bank, or fund family)
 - ▶ j : Observation within group (e.g., firm, branch, fund)
- ▶ Model components:
 - ▶ $y_{i,j}$: outcome (e.g., investment, leverage, return)
 - ▶ $X_{i,j}$: explanatory variable of interest (e.g., policy, treatment)
 - ▶ f_i : unobserved group-level factor (e.g., industry demand, regulation)
 - ▶ $\epsilon_{i,j}$: idiosyncratic error term
- ▶ The key question: what happens if we try to control for f_i without properly including a fixed effect?

The Underlying Model [Part 2]

- ▶ Standard assumptions about the data structure:
 - ▶ N : Number of groups is large; J : Observations per group is small.
 - ▶ $\text{Var}(f_i) = \sigma_f^2$, $\mathbb{E}[f_i] = 0$
 - ▶ $\text{Var}(X_{i,j}) = \sigma_X^2$, $\mathbb{E}[X_{i,j}] = 0$
 - ▶ $\text{Var}(\epsilon_{i,j}) = \sigma_\epsilon^2$, $\mathbb{E}[\epsilon_{i,j}] = 0$
- ▶ X and ϵ are i.i.d. across groups, but may be correlated **within** groups:
 - ▶ Within-group correlation \Rightarrow common shocks.
 - ▶ Across-group independence ensures valid asymptotics.

The Underlying Model [Part 3]

- ▶ Additional assumptions:
 - ▶ $\text{Cov}(f_i, \epsilon_{i,j}) = 0$ — group factors are uncorrelated with idiosyncratic errors.
 - ▶ $\text{Cov}(X_{i,j}, \epsilon_{i,j}) = \text{Cov}(X_{i,j}, \epsilon_{i,-j}) = 0$ — exogeneity of X .
 - ▶ $X_{i,j}$ is exogenous with respect to both its own error term and the error terms of other group members, enabling unbiased and consistent estimation of β in the fixed effects model.
 - ▶ $\text{Cov}(X_{i,j}, f_i) = \sigma_{Xf} \neq 0$ — regressor correlated with group unobservables.
- ▶ Implication:
 - ▶ If we omit f_i , OLS suffers from classic **omitted variable bias**.
- ▶ FE removes f_i through within-group demeaning. AdjY and AvgE only partially do so, leaving residual correlation with f_i . This incomplete adjustment can amplify—or even reverse—the bias.

We Already Know OLS Is Biased

True model: $y_{i,j} = \beta X_{i,j} + f_i + \epsilon_{i,j}$

But OLS estimates: $y_{i,j} = \beta^{OLS} X_{i,j} + u_{i,j}^{OLS}$

- By omitting the group effect f_i , OLS suffers from standard omitted variable bias:

$$\hat{\beta}_{OLS} = \beta + \frac{\sigma_{Xf}}{\sigma_X^2}$$

- Direction and size of bias depend on the covariance between X and f_i .

Adjusted-Y (*AdjY*) Estimation

- ▶ **Idea:** Remove unobserved group effects by demeaning the dependent variable within groups:

$$y_{i,j} - \bar{y}_i = \beta^{AdjY} X_{i,j} + u_{i,j}^{AdjY}$$

- ▶ Group mean:

$$\bar{y}_i = \frac{1}{J} \sum_{k \in i} y_{i,k} = \frac{1}{J} \sum_{k \in i} (\beta X_{i,k} + f_i + \epsilon_{i,k})$$

$$\Rightarrow \bar{y}_i = \beta \bar{X}_i + f_i + \bar{\epsilon}_i$$

- ▶ Some researchers exclude the observation itself or use medians, but bias remains.

Example: *AdjY* Estimation in Practice

- ▶ Example regression:

$$Q_{i,j,t} - \overline{Q}_{i,t} = \alpha + \beta X_{i,j,t} + \epsilon_{i,j,t}$$

- ▶ Variables:

- ▶ $Q_{i,j,t}$: Tobin's Q for firm j in industry i , year t
- ▶ $\overline{Q}_{i,t}$: industry-year mean Q (“industry-adjusted Q”)
- ▶ $X_{i,j,t}$: explanatory variables (e.g., governance, leverage)
- ▶ Often combined with firm or year fixed effects

- ▶ **Question:** Why is *AdjY* still inconsistent?

Why *AdjY* Is Inconsistent

- ▶ Substitute the group mean:

$$\bar{y}_i = \beta \bar{X}_i + f_i + \bar{\epsilon}_i$$

$$y_{i,j} - \bar{y}_i = (\beta X_{i,j} + f_i + \epsilon_{i,j}) - (\beta \bar{X}_i + f_i + \bar{\epsilon}_i)$$

- ▶ The transformation removes f_i in the mean but not in the regressor.
- ▶ When we regress $(y_{i,j} - \bar{y}_i)$ on $X_{i,j}$ instead of $(X_{i,j} - \bar{X}_i)$, the omitted \bar{X}_i induces bias.
- ▶ Hence, *AdjY* omits a relevant group-level term.

AdjY and Omitted Variable Bias

- ▶ The true transformed model is:

$$y_{i,j} - \bar{y}_i = \beta(X_{i,j} - \bar{X}_i) + (\epsilon_{i,j} - \bar{\epsilon}_i)$$

- ▶ But AdjY estimates:

$$y_{i,j} - \bar{y}_i = \beta^{AdjY} X_{i,j} + u_{i,j}^{AdjY}$$

- ▶ Because it omits \bar{X}_i , the estimator is biased:

$$\hat{\beta}_{AdjY} = \beta - \frac{\sigma_{X\bar{X}}^2}{\sigma_X^2}$$

- ▶ With positive $\text{Cov}(X, \bar{X})$ —common under shared industry shocks—bias is typically negative.

Adding a Second Variable, Z

- ▶ Suppose the true model has two regressors:

$$y_{i,j} = \beta X_{i,j} + \gamma Z_{i,j} + f_i + \epsilon_{i,j}$$

- ▶ Maintain previous assumptions and add:
 - ▶ $\text{Cov}(Z_{i,j}, \epsilon_{i,j}) = 0$, $\text{Var}(Z) = \sigma_Z^2$
 - ▶ $\text{Cov}(X, Z) = \sigma_{XZ}$
 - ▶ $\text{Cov}(Z, f_i) = \sigma_{Zf}$
- ▶ AdjY still omits group-level means (\bar{X}_i, \bar{Z}_i) , creating intertwined biases.

AdjY Estimates with Two Variables

- ▶ The biases are now complex:

$$\hat{\beta}_{AdjY} = \beta + \Delta, \quad \hat{\gamma}_{AdjY} = \gamma + \Diamond$$

- ▶ Biases depend on correlations among X, Z, f_i .
- ▶ As Gormley and Matsa (2014) show:
 - ▶ Both coefficients can move in unpredictable directions.
 - ▶ Even sign reversals are possible.

Average Effects ($AvgE$) — Idea

- ▶ Researchers often want to control for unobserved group-level factors (f_i) when fixed effects are unavailable or costly.
- ▶ **Idea:** Include the group mean of the dependent variable as a proxy for f_i :

$$y_{i,j} = \beta^{AvgE} X_{i,j} + \gamma^{AvgE} \bar{y}_i + u_{i,j}^{AvgE}$$

- ▶ Example – Firm profitability regression:

$$ROA_{i,s,t} = \alpha + \beta X_{i,s,t} + \gamma \overline{ROA}_{s,t} + u_{i,s,t}$$

- ▶ $\overline{ROA}_{s,t}$: Average ROA among firms in state s , year t
 - ▶ $X_{i,s,t}$: Firm-level controls (e.g., leverage, size, market share)
- ▶ **Goal:** Use \bar{y}_i to soak up unobserved shocks (f_i) that affect all group members.

Why AvgE Is Problematic

- ▶ The true model:

$$y_{i,j} = \beta X_{i,j} + f_i + \epsilon_{i,j}$$

- ▶ AvgE substitutes a proxy for f_i :

$$\bar{y}_i = \beta \bar{X}_i + f_i + \bar{\epsilon}_i$$

- ▶ Substituting this into the regression gives:

$$y_{i,j} = \beta X_{i,j} + \gamma(\beta \bar{X}_i + f_i + \bar{\epsilon}_i) + u_{i,j}$$

- ▶ **Two problems arise:**

1. **Measurement error:** \bar{y}_i is an imperfect proxy for f_i — it includes $\beta \bar{X}_i + \bar{\epsilon}_i$.
2. **Endogeneity:** controlling for \bar{y}_i removes only the fraction of f_i ; The leftover f_i in the error is still correlated with $X_{i,j}$.

Measurement Error Bias

- ▶ Since

$$\bar{y}_i = f_i + \underbrace{(\beta \bar{X}_i + \bar{\epsilon}_i)}_{\text{measurement error}},$$

\bar{y}_i measures f_i with noise.

- ▶ This creates **measurement error bias**:
 - ▶ As is well known, even classical measurement error causes all estimated coefficients to be inconsistent
- ▶ Bias here is complicated because error can be correlated with both mismeasured variable, f_i , and with $X_{i,j}$.

Summary of OLS, *AdjY*, and *AvgE*

$$\text{True model: } y_{i,j} = \beta X_{i,j} + f_i + \epsilon_{i,j}$$

$$\text{True model: } y_{i,j} - \bar{y}_i = \beta(X_{i,j} - \bar{X}_i) + \epsilon_{i,j} - \bar{\epsilon}_i$$

$$\text{AdjY estimates: } y_{i,j} - \bar{y}_i = \beta^{\text{AdjY}} X_{i,j} + u_{i,j}^{\text{AdjY}}$$

$$\text{AvgE estimates: } y_{i,j} = \beta^{\text{AvgE}} X_{i,j} + \gamma^{\text{AvgE}} \bar{y}_i + u_{i,j}^{\text{AvgE}}$$

- ▶ All three estimators are inconsistent in the presence of unobserved group heterogeneity.
- ▶ *AdjY* and *AvgE* are not necessarily an improvement over OLS.
- ▶ *AdjY* and *AvgE* can yield estimates with opposite signs to the true coefficient.

The Differences Will Matter! Example 1 — Capital Structure

- ▶ Regression model:

$$(D/A)_{i,t} = \alpha + \beta X_{i,t} + f_i + \epsilon_{i,t}$$

- ▶ $(D/A)_{i,t}$: Book leverage for firm i , year t
 - ▶ $X_{i,t}$: Variables affecting leverage (e.g., tangibility, size, profitability)
 - ▶ f_i : Firm fixed effect capturing unobserved, time-invariant factors
- ▶ Data: U.S. firms, 1950–2010, winsorized at 1% tails
- ▶ Goal: Compare how different estimators handle unobserved heterogeneity (f_i)

Capital Structure Regression Results

Dependent Variable: Book Leverage

Variable	OLS	AdjY	AvgE	FE
Fixed Assets / Total Assets	0.270*** (0.008)	0.066*** (0.004)	0.103*** (0.004)	0.248*** (0.014)
Ln(Sales)	0.011*** (0.001)	0.011*** (0.000)	0.011*** (0.000)	0.017*** (0.001)
Return on Assets	-0.015*** (0.005)	0.051*** (0.004)	0.039*** (0.004)	-0.028*** (0.005)
Z-score	-0.017*** (0.000)	-0.010*** (0.000)	-0.011*** (0.000)	-0.017*** (0.001)
Market-to-Book Ratio	-0.006*** (0.000)	-0.004*** (0.000)	-0.004*** (0.000)	-0.003*** (0.000)
Observations	166,974	166,974	166,974	166,974
R^2	0.29	0.14	0.56	0.66

- ▶ Notice how *AdjY* and *AvgE* estimates differ sharply from both OLS and FE.
- ▶ For example, the profitability (ROA) coefficient flips sign under *AdjY*/*AvgE*.
- ▶ Partial corrections for heterogeneity distort inference — bias can even reverse direction.

The Differences Will Matter! Example 2 — Firm Value

- ▶ Regression model:

$$Q_{i,j,t} = \alpha + \beta X_{i,j,t} + f_{j,t} + \epsilon_{i,j,t}$$

- ▶ $Q_{i,j,t}$: Tobin's Q for firm i , industry j , year t
 - ▶ $X_{i,j,t}$: Firm-level determinants of value (e.g., size, R&D, profitability)
 - ▶ $f_{j,t}$: Industry-year fixed effect (controls for sectoral conditions)
- ▶ Data: U.S. manufacturing firms
- ▶ Question: Do OLS, AdjY, AvgE, and FE produce consistent results?

Firm Value Regression Results

Dependent Variable: *Tobin's Q*

Variable	OLS	<i>AdjY</i>	<i>AvgE</i>	FE
Delaware Incorporation	0.100*** (0.036)	0.019 (0.032)	0.040 (0.032)	0.086** (0.039)
Ln(Sales)	-0.125*** (0.009)	-0.054*** (0.008)	-0.072*** (0.008)	-0.131*** (0.011)
R&D Expenses / Assets	6.724*** (0.260)	3.022*** (0.242)	3.968*** (0.256)	5.541*** (0.318)
Return on Assets	-0.559*** (0.108)	-0.526*** (0.095)	-0.535*** (0.097)	-0.436*** (0.117)
Observations	55,792	55,792	55,792	55,792
R^2	0.22	0.08	0.34	0.37

- ▶ *AdjY* and *AvgE* substantially understate the R&D coefficient compared to FE (3.0 vs 5.5).
- ▶ Their partial corrections remove part of the true within-group variation.
- ▶ Overall fit (R^2) confirms this — FE explains far more variation, capturing persistent unobserved factors.

General Implications of the Framework

- ▶ The same logic applies well beyond firm-level panel regressions:
 - ▶ Many “adjusted” estimators implicitly assume the group mean or median removes unobserved heterogeneity.
- ▶ However, any estimator that subtracts off a noisy or endogenous benchmark still suffers from omitted-variable or measurement-error bias.
- ▶ **Examples of biased AdjY-type estimators:**
 - ▶ Subtracting the group **median** or **value-weighted mean** instead of the unobserved fixed effect.
 - ▶ Subtracting the mean outcome of a **matched control sample** (as in diversification-discount studies).
 - ▶ Comparing “adjusted” outcomes before vs. after an event (as in M&A announcement studies).
 - ▶ Using **characteristically adjusted returns** in asset pricing.
- ▶ These adjustments remove some noise but not the unobserved heterogeneity that actually drives bias.

AdjY-Type Estimators in Asset Pricing

- ▶ In empirical asset pricing, researchers often compare returns across portfolios sorted by firm characteristics.
- ▶ Returns are typically “**characteristically adjusted**”:
 - ▶ Subtract the mean return of a benchmark portfolio with similar size, book-to-market, or R&D intensity.
 - ▶ $r_{i,t} - \bar{r}_{\text{benchmark},t}$ is regressed on firm characteristics.
- ▶ **Problem:** This is mathematically equivalent to *AdjY*.
 - ▶ The benchmark mean ($\bar{r}_{\text{benchmark},t}$) is a noisy, endogenous proxy for the unobserved common component (e.g., systematic factor, industry effect).
 - ▶ It does not hold constant the variation in the independent variable across benchmark portfolios.
- ▶ Hence, the adjustment does not eliminate unobserved co-movement — it may even exaggerate it.

Asset Pricing *AdjY* Example — R&D and Stock Returns

- ▶ Example: Firms sorted into quintiles by R&D intensity (R&D/MVE).
- ▶ Researchers compute “characteristically adjusted” yearly returns by subtracting industry-size benchmark means (i.e., an *AdjY* transformation).

Missing	Q1	Q2	Q3	Q4	Q5
-0.012***	-0.033***	-0.023***	-0.002	0.008	0.020***
(0.003)	(0.009)	(0.008)	(0.007)	(0.013)	(0.006)

- ▶ Benchmark portfolios: industry-size matched means of returns.
- ▶ Difference between Q5 and Q1 = 5.3 percentage points.
- ▶ Appears to suggest “high R&D firms outperform.”
- ▶ But since benchmark returns correlate with firm characteristics and unobserved shocks, this inference may be spurious.

Regression Comparison: *AdjY* vs Fixed Effects

<i>Dependent Variable: Yearly Stock Return</i>		
R&D Quintile	<i>AdjY</i> Estimate	FE Estimate
Missing	0.021** (0.009)	0.030*** (0.010)
Quintile 2	0.010 (0.013)	0.019 (0.019)
Quintile 3	0.032*** (0.012)	0.051*** (0.014)
Quintile 4	0.041*** (0.015)	0.068*** (0.018)
Quintile 5	0.053*** (0.011)	0.094*** (0.020)
Observations	144,592	144,592
R^2	0.00	0.40

- ▶ Regression equivalent of the previous “sorts” result.
- ▶ The FE version applies benchmark-period fixed effects to both returns and R&D — conceptually a cleaner “within” estimator.
- ▶ *AdjY* coefficients are consistently smaller in magnitude.
- ▶ R^2 is near zero under *AdjY* but large under FE — indicating that benchmark adjustment misses systematic variation.

What If *AdjY* or *AvgE* Is the True Model?

- ▶ Suppose the data truly follow the *AvgE* structure—where the **group mean outcome** directly affects each member's outcome:

$$y_{i,j} = \beta X_{i,j} + \gamma \bar{y}_i + u_{i,j}$$

- ▶ Then \bar{y}_i itself depends on \bar{u}_i , which includes $u_{i,j}$.
- ▶ This creates a simultaneous relationship: individuals affect the group mean and the group mean affects individuals.
- ▶ This is the classic **reflection problem** (Manski, 1993) — identifying peer or group effects becomes impossible without extra structure or instruments.
- ▶ In this case, **none** of the estimators (OLS, *AdjY*, *AvgE*, or FE) recover the true β . [See Leary and Roberts (2010) for a finance application.]

What If *AdjY* or *AvgE* Is the True Model?

- ▶ Even if the researcher is interested in deviations from the group mean: $(y_{i,j} - \bar{y}_i)$, the *AdjY* estimator is only consistent if $X_{i,j}$ has no effect on $y_{i,j}$.
 - ▶ If $X_{i,j}$ influences $y_{i,j}$, then it must also influence others in the same group ($y_{i,-j}$) through correlated behavior or shared shocks.
 - ▶ Therefore, \bar{X}_i also affects $(y_{i,j} - \bar{y}_i)$, implying:

$$\text{Cov}(X_{i,j}, (y_{i,j} - \bar{y}_i)) \neq 0.$$

- ▶ In short, it is impossible for $X_{i,j}$ to affect $y_{i,j}$ but not the group deviation $(y_{i,j} - \bar{y}_i)$.
- ▶ When true interdependence exists among group members, simple “adjusted” or “demeaned” models confound cause and reflection. Identifying the effect of $X_{i,j}$ requires either instrumental variables or structural modeling of peer interactions.

Summary of Today [Part 1]

- ▶ Our data isn't perfect:
 - ▶ Watch for measurement error.
 - ▶ Watch for survivorship bias.
 - ▶ Be careful about external validity claims.
- ▶ Test that estimates across subsamples are statistically different (if you plan to claim differences).

Summary of Today [Part 2]

- ▶ Don't use $AdjY$ or $AvgE$!
- ▶ Do use fixed effects:
 - ▶ Use benchmark portfolio-period FE in asset pricing rather than characteristically adjusted returns.