Problem Set: Unconfoundedness

BUSS975: Causal Inference in Financial Research

Problem 1 (Covariates, estimands, and the two assumptions)

You want the effect of receiving a bank credit line (D) on a firm's capital expenditure over the following year (Y). You have, or could construct, the following variables:

i. firm size, industry and credit rating, all measured before the line was granted;
ii. demand in the firm's export markets over the prior year, which moves investment but plays no part in the bank's decision;
iii. the amount the firm actually drew on the line;
iv. the number of analysts covering the firm in the year *after* the line was granted, which rises both with credit access and with investment;
v. managerial quality, which drives both the bank's decision and the firm's investment, and which you cannot measure.

(a) Classify each variable as a confounder, a collider, a precision covariate or a mediator, and say whether it belongs in the conditioning set for the *total* effect of the credit line. Exactly one of them is a variable you would **include for some research questions and exclude for others**. Identify it, and state the different question that conditioning on it would answer.

(b) A banking regulator asks: "what would happen to investment if *every* firm had a credit line?" A bank's lending committee asks: "did the lines *we granted* raise investment at the firms that got them?" Name the estimand each question needs. Then state which version of unconfoundedness, and which version of common support, each estimand requires — and explain in one sentence why one of them gets away with the weaker version of both.

(c) Write the identification argument for $ATT(x)$ step by step, starting from

$$ATT(x) = E[Y^1 \mid D{=}1, X{=}x] - E[Y^0 \mid D{=}1, X{=}x],$$

and label each step as either the switching equation or the identifying assumption. On which potential outcome, and in which term, is unconfoundedness used? What weights then turn $ATT(x)$ into the ATT?

(d) Variable (v) cannot be measured, so the design rests on an argument, not a test. Using Imbens's third defence, make the best case you can that unconfoundedness is plausible in this setting — then state the circumstance under which that defence collapses.

Problem 2 (Subclassification, estimands, and common support)

A regulator grants a supervisory waiver (D) to banks. The outcome Y is loan growth (%). Banks fall into three observable strata, and within each stratum the waiver is as good as randomly assigned:

Stratum	N	treated	control	$E[Y \mid D{=}1, X]$	$E[Y \mid D{=}0, X]$
A	400	100	300	14	10
B	400	200	200	20	18
C	200	180	20	31	25

(a) Compute $e(X)$ and the within-stratum effect in each stratum. Using the subclassification weight that defines each estimand, compute the ATE, the ATT and the ATU, and verify that $ATE = \pi \cdot ATT + (1 - \pi) \cdot ATU$, where π is the share treated.

(b) Compute the simple difference in outcomes, $SDO = E[Y \mid D{=}1] - E[Y \mid D{=}0]$, pooling across strata. Decompose the gap $SDO - ATE$ into the **selection bias** term and the **heterogeneous treatment effect bias** term of Lecture 1, and say which dominates — that is, whether the regulator was selecting on levels or on gains.

(c) The ATT exceeds the ATE. Explain why in one sentence, in terms of where the regulator's waivers went.

(d) Suppose stratum A had contained **no** treated banks — all 400 of its banks controls. Which of the ATE, ATT and ATU can you still compute? Compute each one that survives.

(e) Suppose instead stratum C had contained **no** controls — all 200 of its banks treated. Which estimands survive now? Compute each one that survives. State the general rule that (d) and (e) illustrate, and explain why, with ten binary covariates rather than one three-valued one, this situation is the norm rather than a curiosity.

Problem 3 (Matching: the discrepancy, the correction, the standard error)

Four treated banks are matched to the nearest control on a single covariate X (log assets), one neighbour, with replacement. The outcome Y is loan growth (%).

Treated					*Control*				
X_i	2	6	8	10	X_j	1	4	9	30
Y_i	30	44	52	58	Y_j	10	16	26	0

(a) Find each treated bank's match, compute $\hat{\tau}_i = Y_i - Y_{j(i)}$, and report $\widehat{ATT}$. Which control is used twice, and which is never used? What does the unused control tell you about what matching does to a sample?

(b) Compute K_j — the number of times control j is used — for each control, and then $\sum_j K_j(K_j - 1)$. Explain why the Abadie–Imbens variance estimator needs $K_j(K_j - 1)$ rather than K_j.

(c) Take $\hat{\sigma}^2(X_j) = 50$ at the reused control. Compute both terms of

$$\hat{\sigma}^2_{ATT} = \underbrace{\frac{1}{N_T} \sum_{i:D_i=1} \left(\hat{\tau}_i - \widehat{ATT}\right)^2}_{\text{(a) spread of unit effects}} + \underbrace{\frac{1}{N_T} \sum_{j:D_j=0} \frac{K_j(K_j-1)}{M^2}\, \hat{\sigma}^2(X_j)}_{\text{(b) penalty for reuse}},$$

then report $SE(\widehat{ATT}) = \sqrt{\hat{\sigma}^2_{ATT}/N_T}$. By what factor does the reuse term inflate the standard error relative to term (a) alone?

(d) Apply the bias correction of Lecture 3a, §8. Following its Step 2, fit $\hat{\mu}^0(X) = \hat{\alpha} + \hat{\beta}X$ by OLS on the **distinct matched controls only**. Compute $\hat{\tau}_i^{BC} = \hat{\tau}_i - \hat{\beta}\,(X_i - X_{j(i)})$ for each treated bank and report $\widehat{ATT}^{BC}$. Which bank receives the largest correction, and which two factors make it so?

(e) Now refit $\hat{\mu}^0$ on **all four** controls, including the one never used as a match. Report the new $\hat{\beta}$ and $\widehat{ATT}^{BC}$. What happened to the correction, and what does this tell you about how much work the "small" regression is doing? What should you report as a result?

(f) Suppose you had matched on two covariates, log assets and the leverage ratio, where log assets has twenty times the standard deviation of leverage. What would raw Euclidean distance do, and which metric fixes it? Is the fix free?

Problem 4 (Propensity scores and weighting)

(a) The balancing property, $D \perp\!\!\!\perp X \mid e(X)$, holds with no assumption at all; the propensity score theorem needs unconfoundedness. Explain why the first is free, identify the single step in the proof of the second at which unconfoundedness enters, and state what this implies about the evidential value of a balance table.

(b) In your data one covariate has a standardized mean difference of 0.01 and a t-test p-value of 0.83, but a variance ratio of 2.4. What kind of selection produces this pattern? What single change to the score specification would you try, and what statistic would tell you it had worked? Give one reason, beyond this example, why a t-test is the wrong balance diagnostic.

(c) Twelve firms fall into two groups by leverage. Within each group treatment is as good as random, and outcomes do not vary within a group and arm:

Group	firms	treated	control	Y if treated	Y if control
Low leverage (L)	8	2	6	13	10
High leverage (H)	4	3	1	28	20

Compute $e(X)$ in each group, the true ATE, the true ATT and the SDO. Then compute $\widehat{ATE}_{IPW}$ from the Horvitz–Thompson forms of $\widehat{E[Y^1]}$ and $\widehat{E[Y^0]}$, and $\widehat{ATT}_{IPW}$ by reweighting the controls with $\hat{e}/(1-\hat{e})$, using the true group shares as $\hat{e}$. Check what each set of weights sums to, and say what that sum means.

(d) Suppose instead your logit had returned $\hat{e} = 0.20$ for group L and $\hat{e} = 0.80$ for group H. Recompute $\widehat{E[Y^1]}$ both ways: Horvitz–Thompson (divide by N) and Hájek (divide by the sum of the weights). Which is guaranteed to be a weighted average of observed treated outcomes? Which is closer to the truth here — and does normalizing fix a misspecified score?

(e) Now add a third group, V: ten very large firms, **all** of them treated. Which of the ATE, ATT and ATU remain identified? Lecture 3b advises "retreating to the ATT, which only needs overlap in one direction." Does that advice help here? Explain, and say what you would do instead.