BUSS386 Futures and Options

Mid-term Exam — Solutions

Prof. Ji-Woong Chung · Tuesday, October 20, 2026, 15:10–17:10 (120 minutes)

Name: _____

Student ID: _____

Instructions

- Closed book. You may use **one A4 hand-written cheat sheet** (both sides; typed sheets are not permitted) and a **calculator**. Computers, tablets, phones and smart watches are not allowed.
- In Sections A and B, write your answer (T/F, or a–d) clearly next to each question number.
- In Section C, **show your work**. Partial credit is given for correct steps, and a later step that correctly uses a wrong earlier result loses no further points.
- Unless stated otherwise: interest rates are per annum with *continuous compounding*.
- Carry at least **four decimal places** in intermediate steps (e.g., discount factors such as 0.9250, hedge ratios such as 0.7083). Round **final** answers to two decimal places, with rates in percent (e.g., 4.55%).

Section A — True/False

1	2	3	4	5	6	7	8	9	10
T	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**T**

Section B — Multiple Choice

1	2	3	4	5	6	7	8
c	**b**	**a**	**c**	**b**	**c**	**a**	**d**
9	10	11	12	13	14	15	
b	**a**	**d**	**b**	**c**	**a**	**d**	

Section A — True/False **(10 questions × 3 pts = 30 pts)**

A1. A trading desk reports a 1-day 99% VaR of ₩5 billion. Its 1-day 99% Expected Shortfall, stated as a loss amount, must be at least ₩5 billion.

> **True.** VaR is the loss threshold that is exceeded only 1% of the time; ES is the *average* loss in that worst 1%. Every loss in the tail is at least ₩5 billion, so their average is too.

A2. For a given effective annual return, the continuously compounded rate is *higher* than the equivalent annually compounded rate.

> **False.** Continuous compounding is the most frequent compounding, so it needs the *lowest* stated rate (e.g., 14% quarterly = 13.76% c.c. = 14.75% annual).

A3. A callable bond has a higher effective duration than an otherwise identical non-callable bond.

> **False.** The issuer's call caps the price when rates fall, so D_{eff} is *lower* (3.18 vs. 4.38 years in the lecture example).

A4. Because a futures contract is a claim on a physical asset, its price cannot fall below zero.

> **False.** On 20 April 2020 the May WTI contract settled at −\$37.63/bbl: with storage at Cushing nearly full, the cost of taking physical delivery exceeded the value of the oil.

A5. A corn farmer shorts December futures to hedge her harvest. In December the spot price is well above the futures price she sold at. She made a hedging mistake, since she would have earned more without the hedge.

> **False.** The hedge did what it was bought to do: it locked in revenue when she had to commit to the crop. Judging it on the one price path that happened is hindsight; at a lower price the same hedge would have looked brilliant.

A6. If the USD interest rate is higher than the KRW interest rate, the no-arbitrage USD/KRW forward rate (KRW per USD) is below the spot rate.

> **True.** $F_0 = S_0 e^{(r_{\text{KRW}} - r_{\text{USD}})T} < S_0$ when $r_{\text{USD}} > r_{\text{KRW}}$: USD trades at a forward discount (covered interest parity).

A7. If interest rates are positively correlated with the price of the underlying asset, the futures price should be slightly higher than the forward price for the same delivery date.

> **True.** Daily futures gains arrive when rates are high (reinvested at a higher rate), and losses arrive when rates are low (financed cheaply). That advantage makes the long futures more attractive, so the futures price is above the forward price.

A8. Like the old Eurodollar futures, the 3-month SOFR futures contract settles at the *start* of the 3-month period it references.

False. SOFR futures settle at the *end* of the reference period (in arrears), because compounded SOFR is only known once the period is over.

A9. If all zero rates rise after an interest-rate swap is initiated, the swap gains value for the party that *receives* fixed.

False. The receiver is locked into a below-market fixed coupon, so the swap loses value.

A10. Under SOFR compounded in arrears, the floating leg of an interest-rate swap has duration close to zero, so the swap's duration is roughly that of its fixed leg.

True. Nothing is locked in advance under compounding in arrears, so $D_{\text{eff}}^{\text{flt}} \approx 0$ and $D_d^{\text{swap}} \approx N \cdot D_{\text{fix}}$.

Section B — Multiple Choice **(15 questions × 4 pts = 60 pts)**

B1. A Korean investor holds 60% of her portfolio in a KOSPI 200 ETF ($\sigma = 20\%$) and 40% in a USD bond fund ($\sigma = 8\%$). The correlation between the two is $\rho = -0.25$. The portfolio's standard deviation is closest to:

(a) 15.20% (b) 12.42%

(c) 11.62% (d) 13.17%

(c). $\sigma_p^2 = 0.6^2(0.20)^2 + 0.4^2(0.08)^2 + 2(0.6)(0.4)(-0.25)(0.20)(0.08) = 0.0144 + 0.001024 - 0.00192 = 0.013504$, so $\sigma_p = 11.62\%$.

B2. You hold a \$2,000,000 position in an S&P 500 ETF. Daily log returns are normal with mean zero and $\sigma_{\text{daily}} = 1.1\%$. Use $z_{0.01} = 2.326$ (a standard normal variable falls below -2.326 with probability 1%). Using the $\sqrt{T}$ rule, the parametric **10-day 99%** VaR is closest to:

(a) \$51,200 (b) \$161,800

(c) \$114,400 (d) \$511,700

(b). $2.326 \times 0.011 \times \$2{,}000{,}000 \times \sqrt{10} = \$51{,}172 \times 3.162 \approx \$161{,}800$.

B3. A bank quotes 8% per annum with *semiannual* compounding. The equivalent continuously compounded rate is closest to:

(a) 7.84% (b) 8.00%

(c) 8.16% (d) 7.70%

(a). $r_c = 2\ln(1 + 0.08/2) = 2\ln(1.04) = 7.84\%$.

B4. The 1-year and 1.5-year zero rates are 3.2% and 3.5% (continuously compounded). The forward rate for the period from year 1 to year 1.5 is closest to:

(a) 3.35% (b) 3.50%

(c) 4.10% (d) 2.05%

(c). $r_0(1, 1.5) = \dfrac{0.035(1.5) - 0.032(1)}{0.5} = \dfrac{0.0205}{0.5} = 4.10\%$.

B5. A bond is priced at \$95.00 and has a duration of 6.2 years (yield continuously compounded). Using duration only, if the yield *rises* by 30 basis points, the new price is closest to:

(a) \$96.77 (b) \$93.23

(c) \$89.11 (d) \$95.00

(b). $\Delta P \approx -D \cdot P \cdot \Delta y = -6.2 \times 95 \times 0.003 = -\$1.77$, so $P \approx \$93.23$.

B6. You go long one KOSPI 200 futures contract on Day 0 at 405.0. Multiplier ₩250,000 per index point; initial margin ₩10,000,000; maintenance margin ₩7,500,000. Daily settlement prices:

Day	0	1	2	3	4
Settlement	405.0	408.0	402.0	393.0	397.0

Which statement is correct?

(a) Margin call on Day 2; deposit ₩750,000.
(b) Margin call on Day 3; deposit ₩500,000.
(c) Margin call on Day 3; deposit ₩3,000,000.
(d) Margin call on Day 4; deposit ₩2,000,000.

(c). Balances: Day 1 ₩10.75M, Day 2 ₩9.25M, Day 3 ₩7.00M $<$ ₩7.50M $\Rightarrow$ call. The account is restored to the *initial* margin: $10.00 - 7.00 =$ ₩3,000,000.

B7. A Korean airline will buy 4,000,000 gallons of jet fuel in three months and cross-hedges with NYMEX heating-oil futures (42,000 gallons per contract). The standard deviation of monthly changes is $\sigma_S = 0.025$ for the jet-fuel spot price and $\sigma_F = 0.030$ for the heating-oil futures price. The correlation is $\rho = 0.85$. The minimum-variance hedge (rounded to the nearest whole contract) is:

(a) 67 contracts long
(b) 67 contracts short
(c) 79 contracts long
(d) 95 contracts long

(a). $h^* = 0.85 \times 0.025/0.030 = 0.7083$; $0.7083 \times 4{,}000{,}000/42{,}000 = 67.46 \Rightarrow 67$. The airline must *buy* fuel, so the hedge is *long*.

B8. On 1 June, Hyundai Motor expects to receive USD 5,000,000 on 31 August. It shorts KRX September USD/KRW futures at ₩1,395. On 31 August, the spot rate is ₩1,410 and the September futures price is ₩1,418. Hyundai converts its USD at spot and closes out the futures. Its effective KRW rate per USD is:

(a) ₩1,395
(b) ₩1,410
(c) ₩1,433
(d) ₩1,387

(d). Effective rate $= S_{\text{Aug}} + (F_{\text{Jun}} - F_{\text{Aug}}) = 1{,}410 + (1{,}395 - 1{,}418) =$ ₩1,387. Equivalently, it is $F_{\text{Jun}} +$ basis at close $= 1{,}395 + (1{,}410 - 1{,}418)$.

B9. The KOSPI 200 index is at 400 and the multiplier is ₩250,000 per point. KRX requires initial margin of 8% of contract value. A trader buys one contract and commits only the initial margin. If the index falls 2% the next day, her return on the cash committed is:

(a) -2%
(b) -25%
(c) -16%
(d) -8%

(b). Contract value = $400 \times 250{,}000 = ₩100\text{M}$; margin = ₩8M. Loss = 8 pts × 250,000 = ₩2M. Return = $-2/8 = -25\%$ ($= -2\% \times$ leverage of 12.5).

B10. A stock index is at 800. Its dividend yield is $q = 2\%$ and the risk-free rate is $r = 3\%$. The 6-month futures trades at 810. Which arbitrage applies, and what is the approximate profit per index unit at maturity?

(a) Buy the index basket, short the futures, borrow; ≈ 5.99 index points.
(b) Short the index basket, buy the futures, lend; ≈ 5.99 index points.
(c) Short the index basket, buy the futures, lend; ≈ 2.09 index points.
(d) Buy the index basket, short the futures, borrow; ≈ 10.00 index points.

(a). $F_0 = 800\, e^{(0.03-0.02)(0.5)} = 804.01 < 810$, so the futures is overpriced: sell it and carry the index. Profit $= 810 - 804.01 = 5.99$.

B11. The crude-oil futures curve is in *backwardation*: longer-dated contracts trade below spot. Under the cost-of-carry relation $F_0 = S_0\, e^{(r+u-y)T}$, this means:

(a) Storage costs u are unusually high relative to r.
(b) The futures market forecasts that the spot price will rise.
(c) Interest rates must be negative.
(d) The convenience yield y exceeds $r + u$.

(d). $F_0 < S_0 \iff r + u - y < 0$. Holders of the physical commodity value having it on hand. High u pushes toward contango, so (a) is wrong. A storable commodity's forward price is not a forecast, so (b) is wrong.

B12. A trader is short one Treasury bond futures contract with settlement price 92.00. Three bonds are deliverable:

Bond	Quoted price	Conversion factor
A	99.00	1.0500
B	142.00	1.5300
C	119.00	1.2700

Accrued interest on the delivered bond is \$2.50 per \$100 face. The cheapest-to-deliver bond and the cash invoice received by the short (per \$100 face) are:

(a) Bond A; \$99.10
(b) Bond B; \$143.26
(c) Bond B; \$140.76
(d) Bond C; \$119.34

(b). Cost = quoted − 92× CF: A = 2.40, B = 1.24, C = 2.16, so B is the CTD. Invoice $= 92 \times 1.53 + 2.50 =$ \$143.26.

B13. On 1 October, a firm plans to borrow \$50M for 3 months starting in mid-December at SOFR. December 3-month SOFR futures are quoted at 96.20 (\$1M notional; \$25 per basis point per contract). To lock in its borrowing rate, it trades 50 contracts. At the end of the reference period, compounded SOFR turns out to be 4.10%, so the contract settles at 95.90. The position taken and its futures P&L are:

(a) Long; +\$37,500
(b) Short; −\$37,500
(c) Short; +\$37,500
(d) Short; +\$150,000

(c). A borrower loses when rates rise. The futures price = 100− rate falls when rates rise, so the borrower goes *short*. P&L = $(96.20 - 95.90) = 30$ bp × \$25 × 50 = +\$37,500.

B14. In March 2021 the collapse of Archegos Capital cost Credit Suisse about \$5.5 billion. Which feature of the total return swaps (TRS) Archegos used was central to why the risk built up unnoticed?

(a) Archegos held its exposure through TRS instead of owning the shares, so the positions triggered no ownership disclosure and each bank saw only its own slice.
(b) TRS are exchange-traded, so the clearing house absorbed the losses instead of the banks.
(c) A TRS pays only a fixed leg, so Archegos had no exposure to falling stock prices.
(d) TRS required Archegos to post 100% collateral, so the banks' losses came only from operational errors.

(a). Separating economic exposure from legal ownership is what made the build-up invisible. Leverage was the mechanism; invisibility was the problem (Lec 5).

B15. Korean Air enters a 2-year jet-fuel swap. At the end of years 1 and 2, on equal volumes, it pays a fixed price x per barrel and receives the market price. Forward prices are $F_1 = \$90$ and $F_2 = \$98$. Zero rates are 4% (1-year) and 5% (2-year). The fair swap price x is closest to:

(a) \$87.57
(b) \$94.00
(c) \$98.00
(d) \$93.88

(d). Set the fixed leg's PV equal to the forward strip's PV:

$$x = \frac{90e^{-0.04} + 98e^{-0.10}}{e^{-0.04} + e^{-0.10}} = \frac{86.47 + 88.67}{0.9608 + 0.9048} = \frac{175.14}{1.8656} = \$93.88.$$

Section C — Problems **(3 problems × 10 pts = 30 pts)**

C1. Forward on a dividend-paying stock [10 pts]

A 6-month forward is written today on a stock that will pay a \$1 dividend in 2 months and another \$1 dividend in 5 months. The current stock price is \$50 and $r = 6\%$.

(a) **[3 pts]** Compute the no-arbitrage 6-month forward price F_0.

(b) **[5 pts]** *Three months later*, the stock price is \$48 and r is still 6%. Three months remain, and the only remaining dividend is \$1 in 2 months. Compute the current forward price F_t for the same delivery date, and the value f of the original *long* position entered at F_0.

(c) **[2 pts]** In one or two sentences, explain why f is negative even though the contract was struck at the no-arbitrage forward price.

Solution. This problem is identical to **Problem Set 4, Problem 2**. Please refer to the posted Problem Set 4 solutions.

C2. Comparative advantage and the interest-rate swap [10 pts]

Two companies each want to borrow \$100M for 5 years. Quoted rates:

Company	Fixed (5y)	Floating (3M, reset)
AAA Co.	4.0%	SOFR + 0.10%
BBB Co.	5.5%	SOFR + 0.80%

AAA wants to borrow *floating*; BBB wants to borrow *fixed*. A dealer arranges the swap and exchanges SOFR flat with each company on the floating side.

(a) **[3 pts]** Who has the absolute advantage in each market? Who has the comparative advantage in fixed, and who in floating? If AAA borrows fixed at 4% and BBB borrows floating at SOFR + 0.80%, what is the total interest-rate saving available from a swap?

(b) **[5 pts]** The dealer keeps 0.10%, and the rest of the saving is split equally between AAA and BBB. Find the fixed rate the dealer *pays* AAA and the fixed rate the dealer *receives* from BBB. State each company's all-in borrowing cost.

(c) **[2 pts]** What would erode BBB's gain from this arrangement if BBB's credit quality deteriorated during the 5 years? Answer in one or two sentences.

Solution. This problem is identical to **Problem Set 5, Problem 5**. Please refer to the posted Problem Set 5 solutions. Part (a) here covers PS parts (a)–(b); part (b) is PS part (c); part (c) is PS part (d).

C3. Par yield, forward rate and an FRA [10 pts]

The current zero-rate term structure (continuously compounded) is:

Maturity	0.5y	1y	1.5y	2y
Zero rate	3.0%	3.4%	3.7%	3.9%

(a) **[4 pts]** Find the coupon rate (per annum, paid *semiannually*) of a 2-year bond with face value \$100 that is priced at par.

Discount factors: $e^{-0.015} = 0.98511$, $e^{-0.034} = 0.96657$, $e^{-0.0555} = 0.94601$, $e^{-0.078} = 0.92496$; sum $A = 3.82266$. [1.5]

$100 = \frac{c}{2} A + 100(0.92496) \Rightarrow \frac{c}{2} = \frac{7.504}{3.82266} = 1.9630$ [1.5]

$\Rightarrow c = \boxed{3.93\%}$ per annum, paid semiannually. [1]

Accept 3.92% (discount factors carried to four decimals give $7.50/3.8227 = 1.9620$).

(b) **[3 pts]** Compute the forward rate for the period from 18 months to 24 months. Give it with continuous compounding, then convert it to semiannual compounding.

$r_0(1.5, 2) = \frac{0.039(2) - 0.037(1.5)}{0.5} = \frac{0.0225}{0.5} = \boxed{4.50\%}$ c.c. [1.5]

Semiannual: $2\,(e^{0.045 \times 0.5} - 1) = \boxed{4.55\%}$. [1.5]

(c) **[3 pts]** You enter an FRA that pays you a fixed 5.00% (semiannually compounded) and requires you to pay 6-month SOFR on a notional of \$20,000,000, for the 6-month period starting 18 months from now. Using the 2-year zero rate to discount, compute the value of the FRA to you today.

$V = 20{,}000{,}000 \times \frac{0.0500 - 0.04551}{2} \times e^{-0.039 \times 2} = 20{,}000{,}000 \times 0.002245 \times 0.92496 \approx \boxed{\$41{,}530}$. [3]

Accept \$41,300–\$41,700 (rounding of the forward rate). The value is positive because you receive a fixed rate above the 4.55% forward.

— End of Exam —