BUSS386 Midterm Exam — Solutions

Futures and Options

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Section A — True / False

A1. $\boxed{T}$ — $F_0 = S_0\, e^{rT} > S_0$ for any $r > 0$, $T > 0$.

A2. $\boxed{T}$ — by definition of dollar duration.

A3. $\boxed{F}$ — $h^* = \rho \cdot \sigma_S/\sigma_F$; if $\sigma_S > \sigma_F$, h^* can exceed 1.

A4. $\boxed{F}$ — higher coupon *shortens* duration (more weight on earlier cash flows).

A5. $\boxed{F}$ — KOSPI 200 futures are *cash-settled*; no basket of stocks changes hands.

A6. $\boxed{T}$ — Cox–Ingersoll–Ross (1981) result for deterministic rates.

A7. $\boxed{T}$ — SOFR is derived from actual repo transactions, not bank submissions (that was LIBOR's flaw).

A8. $\boxed{F}$ — implied forward rates are no-arbitrage rates; they need not equal the market's unbiased forecast of future spot rates.

A9. $\boxed{T}$ — $e^{0.04} - 1 = 0.04081 = 4.0811\%$.

A10. $\boxed{T}$ — at expiry, $F_T = S_T$, so basis $= 0$.

Section B — Multiple Choice

B1. $\boxed{(b)}$ — CCP becomes the counterparty to both sides; eliminates bilateral credit risk.

B2. $\boxed{(c)}$ — VaR $= 2.326 \times 0.015 \times \$5\text{M} = \$174{,}450$.

B3. $\boxed{(b)}$ — Par bond at 6% annual, 3-year: $D = [1(5.66) + 2(5.34) + 3(89.00)]/100 = 2.83$ years.

B4. $\boxed{(c)}$ — PV(dividend) $= 1 \cdot e^{-0.02} = 0.9802$; $F_0 = (50 - 0.9802)e^{0.04} = 49.02 \cdot 1.0408 = 51.02$.

B5. $\boxed{(b)}$ — Buyer of fuel hedges by going long the futures.

B6. $\boxed{(c)}$ — $F_0 = 500 \cdot e^{(0.04-0.015)\times 0.5} = 500 \cdot e^{0.0125} = 500 \cdot 1.01258 = 506.29$.

B7. $\boxed{(b)}$ — Long SOFR futures gain when SOFR falls (futures price $= 100-$ rate).

B8. $\boxed{(c)}$ — $r_{1,2} = (0.04 \cdot 2 - 0.03 \cdot 1)/(2 - 1) = 5\%$.

B9. $\boxed{(c)}$ — 10 contracts $\times$ 30 bp $\times$ \$25/bp = \$7,500 loss.

B10. $\boxed{(d)}$ — Derivatives don't eliminate *all* risk; basis risk, counterparty risk, and operational risk remain.

Section C — Computational

C1 — Bond pricing and duration

Cash flows: \$6 at $t = 1, 2$; \$106 at $t = 3$. Discount at $y = 5\%$ (c.c.):

$$e^{-0.05} = 0.9512, \quad e^{-0.10} = 0.9048, \quad e^{-0.15} = 0.8607.$$

(a) Price $= 6(0.9512) + 6(0.9048) + 106(0.8607) = 5.71 + 5.43 + 91.23 = \boxed{\$102.37}$.

(b) $D = \dfrac{1(5.71) + 2(5.43) + 3(91.23)}{102.37} = \dfrac{5.71 + 10.86 + 273.70}{102.37} = \boxed{2.84 \text{ years}}$.

(c) $\Delta P/P \approx -D \cdot \Delta y = -2.84 \cdot 0.0025 = -0.71\%$.
New price $\approx 102.37 \cdot (1 - 0.0071) = \boxed{\$101.64}$.

C2 — Forward pricing with a discrete dividend

(a) PV of dividend: $I = 1 \cdot e^{-0.04 \cdot 0.25} = 0.9900$.
Fair forward: $F_0^* = (60 - 0.9900)\, e^{0.04 \cdot 0.75} = 59.01 \cdot e^{0.03} = 59.01 \cdot 1.03045 = \boxed{\$60.81}$.

(b) Market \$63 > \$60.81 ⇒ overpriced. **Strategy:** short the forward, buy the stock, finance with two zeros.

Action	CF at $t = 0$	CF at $t = 0.25$	CF at $t = 0.75$
Buy stock	-60.00	$+1.00$ (div)	S_T
Short forward at \$63	0	0	$63 - S_T$
Sell 3-month zero, notional \$1	$+0.9900$	-1.00	0
Sell 9-month zero, notional \$63	$+61.13$	0	-63.00
Net	$\boxed{+\$2.12}$	0	0

(Using $63 \cdot e^{-0.04 \cdot 0.75} = 63 \cdot 0.9704 = 61.13$.)

(c) Risk-free profit today $= \boxed{\$2.12}$ (equivalently \$2.19 at $t = 0.75$, discounted by $e^{-0.03}$).

C3 — Margin account and daily settlement

Multiplier $M =$ ₩250,000. Daily P&L $= M \cdot (F_t - F_{t-1})$.

(a) Daily P&L (multiplier × index change):

- Day 1: $250{,}000 \times (405 - 400) = +1{,}250{,}000$ ₩.
- Day 2: $250{,}000 \times (398 - 405) = -1{,}750{,}000$ ₩.
- Day 3: $250{,}000 \times (388 - 398) = -2{,}500{,}000$ ₩.
- Day 4: $250{,}000 \times (392 - 388) = +1{,}000{,}000$ ₩.

(b) End-of-day balances (before any margin top-up):

Day	0	1	2	3	4
Balance (₩M)	10.00	11.25	9.50	**7.00**	(see (c))

(c) Margin call triggered on **Day 3**: balance ₩7.00M falls below maintenance ₩7.50M.
Required deposit to restore initial margin: 10M − 7M = $\boxed{3{,}000{,}000}$.

C4 — USD/KRW forward and arbitrage

(a) $F_0^* = 1{,}400 \cdot e^{(0.020-0.045)\cdot 0.5} = 1{,}400 \cdot e^{-0.0125} = 1{,}400 \cdot 0.9876 = \boxed{1{,}382.6 \text{ per USD}}$.

(b) Market 1,400 > 1,382.6 ⇒ forward overpriced. **Strategy:** short forward, buy USD spot, finance with KRW borrow.

Action (Korean investor)	CF today (₩)	CF at $T = 0.5$ (₩)
Buy $e^{-0.0225} = 0.9778$ USD at ₩1,400	−1,368.9	USD deposit → 1 USD → S_T
Short USD/KRW forward (1 USD at ₩1,400)	0	$1{,}400 - S_T$
Borrow ₩1,368.9 at $r = 2.0\%$, 6 months	+1,368.9	$-1{,}368.9 \cdot e^{0.01} = -1{,}382.6$
Net	0	$\boxed{+17.4}$

(c) Holding USD pays a higher interest rate ($r_{\text{USD}} > r_{\text{KRW}}$). For no-arbitrage between (i) holding USD and earning r_{USD} vs. (ii) holding KRW at r_{KRW} and converting via forward, the forward USD price must *decline* (USD at discount) to offset the higher USD yield.

C5 — Valuing a live forward

(a) $F_0 = 40 \cdot e^{0.05} = 40 \cdot 1.0513 = \boxed{\$42.05}$.

(b) Today, 6 months remain: $F_t = 45 \cdot e^{0.05\cdot 0.5} = 45 \cdot 1.0253 = \boxed{\$46.14}$.

(c) Value of original long position:

$$f = (F_t - F_0)\, e^{-r(T-t)} = (46.14 - 42.05) \cdot e^{-0.025} = 4.09 \cdot 0.9753 = \boxed{\$3.99}.$$

Verification via S_t formula:

$$f = S_t - F_0\, e^{-r(T-t)} = 45 - 42.05 \cdot 0.9753 = 45 - 41.01 = \$3.99. \quad \checkmark$$