BUSS386 Problem Set 12

Volatility

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Problem 1 — Historical, EWMA, and GARCH on the same returns

The KOSPI 200 daily closing levels (stylized): 360.0, 363.6, 360.0, 367.2, 363.5, 369.8.

(a) Compute the 5 daily log returns R_t and their squares.

(b) Compute the historical daily volatility (zero-mean estimator) and annualize using $h = 252$.

(c) Compute the EWMA daily volatility with $w = 0.94$ (RiskMetrics convention). Annualize.

(d) Set up a GARCH(1,1) with $\alpha = 0.05$, $\beta = 0.94$, σ_∞^2 matched to the historical estimate. Compute σ_2 and σ_3 (the two next-day variance forecasts).

Problem 2 — Implied volatility inversion

A 3-month European call on a non-dividend stock with $S_0 = 100$, $K = 100$, $r = 4\%$ c.c. trades at \$5.00. Find the implied volatility.

(a) Compute the BSM call price for $\sigma = 20\%$. Is the market price above or below it?

(b) Compute the BSM call price for $\sigma = 25\%$. Bracket the IV.

(c) Use linear interpolation between (a) and (b) to estimate the implied volatility to the nearest 0.5%.

Problem 3 — The vol risk premium and gamma scalping

Over the last 25 years on the S&P 500, the mean 1-month VIX has been ∼19%, and the mean 1-month realized vol has been ∼15%. A long-gamma delta-hedged position has approximate cumulative P&L per unit notional

$$\text{P\&L} \;\propto\; \tfrac{1}{2}\Gamma S_0^2(\sigma_{\text{real}}^2 - \sigma_{\text{imp}}^2)\,T.$$

(a) Compute the sign of the average gamma-scalper's P&L over a long horizon. Is the strategy expected to make or lose money on average?

(b) In one sentence, why does the typical *short*-vol strategy (covered call, JEPI, Korean ELS issuer) make money on average?

(c) In one sentence, what is the catch?

Problem 4 — Smile-adjusted equity put

KOSPI 200 spot $S_0 = 360$, $r = 3\%$ c.c., $q = 1.5\%$ (continuous dividend), $T = 0.5$. The market exhibits the equity skew with the following IVs:

K	324	342	360	378
IV	24%	21%	18%	16%

Price the $K = 342$ European put two ways:

(a) With the flat ATM IV of 18%.

(b) With the smile-adjusted IV of 21%.

(c) Compute the smile premium (b − a) per index point and per contract (multiplier ₩ 250,000).

Problem 5 — Surface interpolation

The KOSPI 200 IV grid:

$T \setminus K/S_0$	0.95	1.00	1.05	1.10
3 months	21%	18%	16%	15%
6 months	20%	17.5%	16%	15.5%
12 months	19%	17%	16.5%	16.5%

(a) Read off the IV for $K/S_0 = 1.05$, $T = 6$ months.

(b) Use linear interpolation in T to estimate the IV for $K/S_0 = 1.05$, $T = 9$ months.

(c) Use bilinear interpolation to estimate the IV at $K/S_0 = 1.03$, $T = 6$ months.

(d) Looking at the surface, is the skew *steeper* at 3 months or 12 months? In one sentence, why is this typical?

Problem 6 — Variance swap

A trader buys a 3-month variance swap on the KOSPI 200 with strike $K_{\text{var}} = 0.0324$ (corresponding to 18% vol) and notional ₩ 1B.

(a) If realized variance over the 3 months is 0.0400 (20% vol), compute the payoff.

(b) If realized variance is 0.0225 (15% vol), compute the payoff.

(c) In one sentence, why are variance swaps preferred to volatility swaps in practice?