BUSS386 Problem Set 11 — Solutions

Option Greeks

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Problem 1 — Computing all five Greeks

$S_0 = 100$, $K = 100$, $r = 0.04$, $\sigma = 0.25$, $T = 0.5$. $\sigma\sqrt{T} = 0.17678$.

(a) $d_1 = \dfrac{0 + (0.04 + 0.03125)(0.5)}{0.17678} = \dfrac{0.035625}{0.17678} = \boxed{0.2015}$. $d_2 = 0.2015 - 0.1768 = \boxed{0.0247}$.

(b) $N(d_1) = 0.5799$, $N(d_2) = 0.5099$, $N'(d_1) = 0.3909$.

- $\Delta = N(d_1) = \boxed{0.580}$.
- $\Gamma = \dfrac{N'(d_1)}{S\sigma\sqrt{T}} = \dfrac{0.3909}{100 \cdot 0.25 \cdot 0.7071} = \boxed{0.0221}$.
- $\Theta = -\dfrac{SN'(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT}N(d_2) = -6.911 - 1.999 = \boxed{-8.91}/\text{yr} = \boxed{-0.0244}/\text{day}$.
- $\nu = S\sqrt{T}N'(d_1) = 100 \cdot 0.7071 \cdot 0.3909 = 27.64 \Rightarrow \boxed{0.276}$ per 1% vol.
- $\rho = KTe^{-rT}N(d_2) = 100 \cdot 0.5 \cdot 0.9802 \cdot 0.5099 = 24.99 \Rightarrow \boxed{0.250}$ per 1% rate.

(c) Put Greeks (same strike/maturity): $\Delta_p = N(d_1) - 1 = \boxed{-0.420}$; $\Gamma_p = \Gamma = 0.0221$ (same); $\nu_p = \nu = 0.276$ (same); $\Theta_p = -\dfrac{SN'(d_1)\sigma}{2\sqrt{T}} + rKe^{-rT}N(-d_2) = -6.911 + 1.921 = \boxed{-4.99}/\text{yr}$; $\rho_p = -KTe^{-rT}N(-d_2) = \boxed{-0.240}$ per 1% rate.

Problem 2 — Delta vs probability of exercise

(a) $\Delta = N(d_1) = \boxed{0.580}$; RN exercise probability $= N(d_2) = \boxed{0.510}$.

(b) They differ because $d_1 - d_2 = \sigma\sqrt{T} = 0.177 > 0$, so $N(d_1) > N(d_2)$. Delta exceeds the RN probability of finishing ITM.

(c) As $T \to 0$ with $S = K$ (ATM): $d_1, d_2 \to 0$, so both $N(d_1), N(d_2) \to \boxed{0.5}$. The gap $\sigma\sqrt{T} \to 0$.

Problem 3 — Delta hedging a book

50,000 calls, $\Delta = 0.40$, $\Gamma = 0.05$.

(a) Position delta $= 50{,}000 \times 0.40 = 20{,}000$. $\boxed{\text{Short 20,000 index units}}$.

(b) Index 360 $\rightarrow$ 365, $\Delta S = 5$. New delta/option $\approx 0.40 + 0.05(5) = 0.65$. Position delta $= 50{,}000 \times 0.65 = 32{,}500$. You are short 20,000 but need short 32,500, so $\boxed{\text{sell 12,500 more units at 365}}$ — "sell high."

(c) $\boxed{\text{Long gamma}}$ (long options). The round trip (sell high now, buy back lower later) **makes money**.

Problem 4 — Gamma scalping

Long 1 call, $\Delta = 0.50$, $\Gamma = 0.05$, $S_0 = 200$, short 0.50 shares.

(a) $S \rightarrow 210$ ($\Delta S = 10$): new delta $\approx 0.50 + 0.05(10) = 1.00$. Short 0.50, need short 1.00 $\Rightarrow$ $\boxed{\text{sell 0.50 shares at \$210}}$.

(b) $S \rightarrow 200$ ($\Delta S = -10$): new delta $\approx 1.00 - 0.05(10) = 0.50$. Short 1.00, need short 0.50 $\Rightarrow$ $\boxed{\text{buy back 0.50 shares at \$200}}$.

(c) Sold 0.50 sh 210, bought 0.50 sh 200: profit $= 0.50 \times (210 - 200) = \boxed{\$5.00}$.

(d) Over the option's life the strategy is profitable iff $\sigma_{\text{real}} > \sigma_{\text{imp}}$ — i.e., the gamma scalp out-earns the theta you pay.

Problem 5 — The Θ–Γ identity

$\Gamma = -20$, $S = 100$, $\sigma = 0.20$, $r = 0.05$, $V = -50{,}000$ (a short-option portfolio is a liability, $V < 0$).

(a) $\Theta = rV - \frac{1}{2}\sigma^2 S^2 \Gamma = 0.05(-50{,}000) - \frac{1}{2}(0.04)(10{,}000)(-20) = -2{,}500 - (-4{,}000) = \boxed{+1{,}500}$/yr.

(b) $\boxed{\text{Short gamma}}$ ($\Gamma < 0$), $\boxed{\text{positive theta}}$ ($\Theta > 0$): the portfolio earns time decay but carries curvature risk — it loses on large moves in either direction.

(c) A $\boxed{\text{short straddle}}$ (or short strangle, or short iron condor) produces $\Gamma < 0$, $\Theta > 0$: collect premium, bleed on big moves.

Problem 6 — Delta-gamma hedging

Short a long-dated call: $\Delta = 0.60$, $\Gamma = 0.08$. Traded option: $\Delta_1 = 0.55$, $\Gamma_1 = 0.25$. Portfolio $\Pi = -C + NS + N^C C_1$.

(a) Gamma-neutral: $-\Gamma + N^C \Gamma_1 = 0 \Rightarrow N^C = \dfrac{\Gamma}{\Gamma_1} = \dfrac{0.08}{0.25} = \boxed{0.32}$ (long 0.32 of the traded option).

(b) Delta-neutral: $-\Delta + N + N^C \Delta_1 = 0 \Rightarrow N = \Delta - N^C \Delta_1 = 0.60 - 0.32(0.55) = 0.60 - 0.176 = \boxed{0.424}$ shares (long).

(c) A share has *zero* gamma, so it cannot offset the call's curvature. Only a convex instrument (another option) can hedge gamma.