BUSS386 Problem Set 11

Option Greeks

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Problem 1 — Computing all five Greeks

A non-dividend European call: $S_0 = 100$, $K = 100$, $r = 4\%$ c.c., $\sigma = 25\%$, $T = 0.5$.

(a) Compute d_1 and d_2.

(b) Compute Δ, Γ, Θ (per year and per calendar day), ν (per 1% vol), and ρ (per 1% rate).

(c) State the corresponding Greeks for the European *put* with the same strike and maturity (use the relationships, not a full recomputation).

Problem 2 — Delta vs probability of exercise

Using the option in Problem 1:

(a) Compute the call's delta $N(d_1)$ and the risk-neutral probability of exercise $N(d_2)$.

(b) Explain in one sentence why they differ, and which one is larger.

(c) As $T \to 0$ with the option ATM, what do $N(d_1)$ and $N(d_2)$ each approach?

Problem 3 — Delta hedging a book

You are long 50,000 KOSPI 200 calls (each on 1 index unit), $\Delta = 0.40$, $\Gamma = 0.05$ per option. The index is at 360; multiplier ₩250,000 per point.

(a) How many index units must you short to be delta-neutral?

(b) The index rises to 365. Estimate the new position delta using gamma. What rebalancing trade is required, and is it "buy high" or "sell high"?

(c) Are you long or short gamma? In one sentence, does your rebalancing make or lose money on a round trip?

Problem 4 — Gamma scalping

You are long 1 call with $\Delta = 0.50$, $\Gamma = 0.05$, on a stock at $S_0 = 200$. You delta-hedge by shorting 0.50 shares.

(a) The stock rises to \$210. Estimate the new delta. What share trade restores delta-neutrality, and at what price?

(b) The stock falls back to \$200. Estimate the new delta and the share trade, at what price?

(c) Compute the scalping profit from this round trip (ignore theta).

(d) In one sentence, what determines whether the full strategy (including theta) is profitable over the option's life?

Problem 5 — The Θ–Γ identity

A delta-neutral portfolio of options on a non-dividend stock has $\Gamma = -2{,}000$ (per the stock's price units), with $S = 100$, $\sigma = 20\%$, $r = 5\%$. The portfolio value is $V = \$40{,}000$.

(a) Use $\Theta + \frac{1}{2}\sigma^2 S^2 \Gamma = rV$ to compute Θ.

(b) Is the portfolio long or short gamma? Is its theta positive or negative? Interpret the sign in one sentence.

(c) In one sentence, what kind of trade (e.g., short straddle, long straddle, iron condor) would produce $\Gamma < 0$, $\Theta > 0$?

Problem 6 — Delta-gamma hedging

You are short a long-dated call with $\Delta = 0.60$, $\Gamma = 0.08$. A traded short-dated call has $\Delta_1 = 0.55$, $\Gamma_1 = 0.25$. The stock is at \$50.

(a) How many units N^C of the traded option make the position gamma-neutral?

(b) Given N^C, how many shares N make it delta-neutral?

(c) In one sentence, why can't you hedge gamma with shares alone?