BUSS386 Problem Set 9

Binomial Trees

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Problem 1 — One-step binomial replication

A non-dividend stock trades at $S_0 = 50$. Over the next 6 months it will either rise to \$60 or fall to \$45. $r = 4\%$ c.c.

(a) Identify u, d, and compute the risk-neutral probability p.

(b) Price a 6-month European call with $K = 50$ via the replicating portfolio. Show Δ, B, and f_0.

(c) Verify by computing $f_0 = e^{-rT}[pf_u + (1-p)f_d]$ directly.

(d) In the same tree, price the European put with $K = 50$. Confirm put-call parity.

Problem 2 — Two-step European put on KOSPI 200

KOSPI 200 spot $S_0 = 360$. Over each 3-month step the index moves up by 5% or down by 5% ($u = 1.05$, $d = 0.95$). $r = 3\%$ c.c. Consider a 6-month ($N = 2$) European put with $K = 360$. Multiplier ₩250,000 per index point.

(a) Compute the RN probability p for one step.

(b) Compute the index level and put payoff at each terminal node.

(c) Backward-induct to find f_0 in index points.

(d) Convert to ₩ per contract.

Problem 3 — Three-step European call

$S_0 = K = \$30$, $\Delta t = 1$, $u = 1.10$, $d = 0.90$, $r = 5\%$ c.c., $N = 3$.

(a) Compute p and the four terminal payoffs.

(b) Use the closed-form N-step formula to price the call.

(c) Verify by backward induction (one column at a time).

Problem 4 — American put with early-exercise wedge

Same parameters as Problem 3, but consider an American *put* with $K = 30$.

(a) Compute the put payoff at each terminal node.

(b) Backward-induct with the early-exercise check at every interior node. State which nodes (if any) are exercised early.

(c) Compute f_0.

(d) Repeat (b)–(c) as the corresponding European put. Compute the early-exercise premium $P^{\text{Am}} - P^{\text{Eur}}$.

Problem 5 — American call with discrete dividend

A stock with $S_0 = 100$ pays a \$3 cash dividend just before the step-1 node. $u = 1.20$, $d = 0.85$, $\Delta t = 0.5$, $N = 2$, $r = 4\%$ c.c. Consider an American call with $K = 100$.

(a) Compute the ex-dividend prices at step 1 and the terminal prices.

(b) Compute p and terminal payoffs.

(c) At the up node at step 1, compare "exercise now (cum-dividend)" to "wait." Which dominates?

(d) Compute f_0.

(e) In one sentence, why does the dividend make early exercise potentially optimal?

Problem 6 — Calibrating u, d, p from volatility

Suppose the annualized volatility of a stock's daily log returns is $\sigma = 25\%$. You want to build a 4-step tree for a 1-year option, $\Delta t = 0.25$. $r = 4\%$, $q = 1\%$ (continuous dividend).

(a) Compute u, d, p.

(b) For $S_0 = 100$, list the five terminal stock prices.

(c) Use the closed-form N-step formula to price a European call with $K = 100$.

Problem 7 — Merton model / firm valuation

Firm assets $V_0 = ₩\,500$B with volatility $\sigma_V = 20\%$. Debt face $L = ₩\,400$B due in 2 years. $r = 3\%$ c.c. Use a 2-step tree ($\Delta t = 1$, $u = e^{0.20}$, $d = e^{-0.20}$).

(a) Compute terminal asset values and equity payoffs $E_T = \max(V_T - L, 0)$.

(b) Price equity today by backward induction.

(c) Compute the value of debt today and its implied yield.

(d) Compute the credit spread vs the risk-free rate.

(e) In one sentence, what is the real-world Korean application of this calculation (chaebol, conglomerate, CB issuance)?