BUSS386 Problem Set 8 — Solutions

Trading Strategies Involving Options

Prof. Ji-Woong Chung

Problem 1 — Synthetic positions

$Ke^{-rT} = 50 \cdot e^{-0.02} = 50 \cdot 0.9802 = 49.010$.

(a) Parity LHS $= c + Ke^{-rT} = 4.20 + 49.010 = 53.21$.
Parity RHS $= p + S_0 = 2.80 + 51 = 53.80$.
Discrepancy $= -0.59 \Rightarrow$ $\boxed{\text{parity off by \$0.59}}$ (within typical bid-ask).

(b) Synthetic short stock = short call + long put + short bond face K.
Cost today $= -c + p - Ke^{-rT} = -4.20 + 2.80 - 49.010 = \boxed{-\$50.41}$.
(We *receive* \$50.41 today, like selling at \$50.41.)

(c) Synthetic short avoids short-sale constraints, hard-to-borrow fees, and KRX uptick rules; useful when actual shorting is restricted or costly.

Problem 2 — Covered call on KOSPI 200 ETF

(a) Premium = 1,000× ₩500 = $\boxed{500{,}000}$.

(b) Portfolio value at T = stock value + short call payoff + premium kept.

S_T (₩)	Stock value	Short call payoff	Total value (incl. premium)
38,000	38,000,000	0	38,500,000
40,000	40,000,000	0	40,500,000
41,000	41,000,000	0	$\boxed{41{,}500{,}000}$
42,000	42,000,000	−1,000,000	41,500,000
45,000	45,000,000	−4,000,000	41,500,000

(c) Max value = ₩41,500,000 at any $S_T \geq$ ₩41,000.

(d) Original portfolio value ₩40,000,000. Premium covers a price drop down to ₩39,500. Breakeven $S_T = \boxed{₩39{,}500}$.

(e) Trade-off: cap your upside at ₩41,500,000 in exchange for a ₩500,000 cushion on the downside.

Problem 3 — Zero-cost collar

(a) Put premium paid \$3.50 = call premium received \$3.50 $\Rightarrow$ $\boxed{\text{net zero cost}}$.

(b) Hedged share value $= S_T + \max(70 - S_T, 0) - \max(S_T - 95, 0)$:

S_T	Per share	×1,000,000 (\$)
60	70	70,000,000
70	70	70,000,000
80	80	80,000,000
95	95	95,000,000
110	95	95,000,000

(c) Min = $\boxed{\$70{,}000{,}000}$ (floor at K_p), max = $\boxed{\$95{,}000{,}000}$ (ceiling at K_c).

(d) Selling the stock outright would trigger capital gains tax and signal lack of confidence to the market; the collar achieves downside protection without sale.

Problem 4 — Bear spread with calls

(a) Bear with calls: $\boxed{\text{sell low-strike, buy high-strike}}$. Net premium $= 8 - 3 = +\$5$ (credit received).

(b) Profit = net payoff + \$5:

S_T	Short $K = 45$ call	Long $K = 55$ call	Net payoff	Profit
40	0	0	0	$\boxed{+5}$
45	0	0	0	+5
50	−5	0	−5	0
55	−10	0	−10	−5
60	−15	5	−10	$\boxed{-5}$

(c) Max profit $= +\$5$ (when $S_T \leq 45$); max loss $= -\$5$ (when $S_T \geq 55$). Breakeven $S_T = \boxed{\$50}$.

Problem 5 — Butterfly arbitrage

(a) Long butterfly: long $K = 90$ + long $K = 110 - 2$ short $K = 100$. Net premium $= 13+3-2\cdot 7 =$ $\boxed{+\$2}$ (cost).

(b) Payoff at expiry:

S_T	$\max(S_T - 90, 0)$	$\max(S_T - 110, 0)$	$-2\max(S_T - 100, 0)$	Net
85	0	0	0	0
95	5	0	0	5
100	10	0	0	10
105	15	0	−10	5
115	25	5	−30	0

(c) Cost is positive (\$2), payoff is non-negative everywhere, peaks at \$10 at $S_T = 100$. $\boxed{\text{No arbitrage — convexity}}$

Problem 6 — Straddle vs strangle

(a) Straddle cost $= 4 + 4 = \boxed{\$8}$. Strangle cost $= 2 + 2 = \boxed{\$4}$.

(b) Breakevens:

- Straddle: $100 \pm 8 = \{92, 108\}$.
- Strangle: lower $= 95 - 4 = 91$; upper $= 105 + 4 = 109$.

(c) Profit table:

S_T	Straddle profit	Strangle profit
85	$15 - 8 = +7$	$10 - 4 = +6$
92	$8 - 8 = 0$	$3 - 4 = -1$
100	$0 - 8 = -8$	$0 - 4 = -4$
108	$8 - 8 = 0$	$3 - 4 = -1$
115	$15 - 8 = +7$	$10 - 4 = +6$

(d) Small move (± 3): both lose, but the strangle loses less (\$-4 vs \$-8 at $S_T = 100$) — it costs less to set up. $\boxed{\text{Strangle better for small moves.}}$
Large move ($\pm 10+$): both profit; the straddle profits more (its kink is closer to the action). $\boxed{\text{Straddle better for large moves.}}$

Problem 7 — Iron condor on SPX 0DTE

(a) Four legs: $\boxed{\text{long 5,750 put + short 5,780 put + short 5,820 call + long 5,850 call}}$.

(b) Max profit = net credit $= \$8 \cdot 100 = \boxed{\$800}$ per contract, achieved if $5{,}780 \leq S_T \leq 5{,}820$.
Max loss = spread width minus credit $= (5{,}780 - 5{,}750) - 8 = 22$ index points $= 22 \cdot 100 = \boxed{\$2{,}200}$, achieved if $S_T \leq 5{,}750$ or $S_T \geq 5{,}850$.

(c) The risk-reward is \$800 won vs \$2,200 lost — the seller wins more often than loses but each loss is larger. The strategy is profitable only if implied vol over-estimates realized vol (the volatility risk premium).

(d) No. If realized $\sigma >$ implied, the seller loses on average. 0DTE iron condors are a bet that the market *over-prices* realized variance — which is empirically true on average but with painful tails (the XIV trade in disguise).

Problem 8 — Box spread financing

(a) Long K_1 call + short K_2 call + short K_1 put + long K_2 put. At expiry:

$$(S_T - K_1)^+ - (S_T - K_2)^+ + (K_2 - S_T)^+ - (K_1 - S_T)^+ = K_2 - K_1 = \boxed{\$200}.$$

(b) Implied rate r^* from $197.50 = 200 \cdot e^{-r^* \cdot 0.5}$:

$$r^* = -\frac{1}{0.5} \ln\left(\frac{197.50}{200}\right) = -2\ln(0.9875) = 2 \cdot 0.01258 = \boxed{2.52\%}.$$

(c) $r^* = 2.52\% < 4\%$ market rate $\Rightarrow$ box yields *less* than Treasuries $\Rightarrow$ $\boxed{\text{box is rich (overpriced)}}$ relative to risk-free.

(d) Box rates differ from Treasuries because they reflect (i) market-maker credit, not Treasury credit; (ii) capital costs of holding the four option legs; (iii) early-assignment risk on American options; and (iv) supply/demand imbalances when retail uses boxes for cheap leverage.