BUSS386 Problem Set 7

Properties of Options: Bounds and Parity

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Problem 1 — Six determinants

Consider a European call and a European put with the same strike and maturity on the same non-dividend stock.

(a) Fill in the sign (↑, ↓, or 0) of the partial derivative of c and p with respect to each factor:

Factor ↑	European call c	European put p
Spot S_0		
Strike K		
Time to expiry T		
Volatility σ		
Risk-free rate r		
Dividends D		

(b) Explain in one sentence why higher σ raises *both* c and p.

(c) Explain in one sentence why an American put can be worth strictly more than the corresponding European put, but an American call on a non-dividend stock cannot.

Problem 2 — Bounds in the strike

Consider European calls on the same underlying and maturity at three strikes $K_1 = 95, K_2 = 100, K_3 = 105$. Quotes: $c(95) = 7.50$, $c(100) = 5.00$, $c(105) = 2.20$. Assume $r = 4\%$ c.c., $T = 0.25$ yr.

(a) Is the monotonicity condition $c(K_1) \geq c(K_2) \geq c(K_3)$ satisfied?

(b) Is the slope condition $0 \leq c(K_1) - c(K_2) \leq (K_2 - K_1)e^{-rT}$ satisfied?

(c) Is the convexity condition $c(K_2) \leq \frac{1}{2}[c(K_1) + c(K_3)]$ satisfied? If not, construct the butterfly arbitrage.

Problem 3 — Put-call parity check

SPX is at 5,800. A 3-month European call with $K = 5{,}800$ trades at \$140. A 3-month European put with the same K trades at \$80. The 3-month risk-free rate is $r = 4\%$ (c.c.); assume no dividends.

(a) State the put-call parity identity.

(b) Verify (or reject) parity for these quotes. Compute the discrepancy.

(c) If parity is violated by more than the bid-ask spread, sketch the arbitrage strategy (which side is rich, which is cheap, and which portfolio do you buy/sell?). *One short paragraph; no detailed cash flow table needed.*

Problem 4 — Lower-bound arbitrage on a European put

A European put on a non-dividend stock has $K = \$40$, $T = 0.5$ yr, $r = 5\%$ c.c. The stock is at $S_0 = \$37$ and the put is quoted at \$1.00.

(a) Compute the no-arbitrage lower bound on the put price.

(b) Show the arbitrage strategy in a 3-row cash-flow table (today, $S_T \geq K$, $S_T < K$).

(c) Compute the arbitrage profit today.

Problem 5 — Forward-style parity on KOSPI 200

KOSPI 200 spot $S_0 = 360$. A 6-month European call with $K = 360$ trades at 10.50 index points; the European put at the same K trades at 6.20. $r = 3\%$ c.c., dividend yield $q = 1.5\%$. Multiplier = ₩ 250,000 per point.

(a) Compute the 6-month forward price F_0 of the KOSPI 200.

(b) State the forward-style parity and verify it for the given quotes.

(c) If parity is violated, identify which side is rich and sketch the arbitrage trade.

(d) Compute the arbitrage profit per contract in ₩ (ignore bid-ask).

Problem 6 — Discrete-dividend parity

A 1-year European call on a stock with $K = \$50$ trades at \$6. The stock is at $S_0 = \$52$ and will pay a \$1.50 dividend in 6 months. $r = 4\%$ c.c.

(a) Compute the PV of dividends D.

(b) Use put-call parity to find the no-arbitrage price of a 1-year European put with the same strike.

(c) Without recomputing, by how much would the put price change if the dividend doubled to \$3.00?

Problem 7 — Early exercise of American options

(a) State the result for early exercise of an American call on a non-dividend-paying stock and prove it using put-call parity for European options.

(b) Give one concrete situation (state the parameter values you choose) in which early exercise of an American call *with* a discrete dividend would be optimal just before ex-dividend. Verify your numerical claim against the heuristic rule

$$D \;>\; K(1 - e^{-r(T-t_d)}) + p_{t_d}.$$

(c) In one sentence, explain why early exercise of an American put can be optimal even with no dividends, in terms of the interest forgone vs the time value of waiting.