BUSS386 Problem Set 5 — Solutions

Swaps

Prof. Ji-Woong Chung

Note. Problem Set 5 is for *self-study only* — no submission required. These solutions are posted to help you check your work.

Problem 1 — Cheapest-to-deliver & futures invoice (10 pts)

(a) Cost of delivery = quoted price − settlement × CF:

Bond	Quoted	92.00 × CF	Cost	
A	99.00	96.60	2.40	
B	142.00	140.76	$\boxed{1.24}$	CTD
C	119.00	116.84	2.16	

Bond B is the CTD.

(b) Invoice cash = settlement × CF + accrued interest $= 92.00 \times 1.5300 + 2.50 = 140.76 + 2.50 =$ $\boxed{\$143.26}$ per \$100 face.

(c) The CF is defined as the price of the deliverable bond *at the benchmark yield* (6% for US T-bond futures). Without a fixed benchmark, CFs would be ambiguous and the invoice scaling would not put bonds with different coupons on a comparable footing.

Problem 2 — SOFR futures hedge (15 pts)

(a) **Long** the futures. SOFR futures price = 100 − rate, so *long* gains when rates *fall*. The depositor loses when rates fall (lower interest), so a long futures position offsets that risk.

(b) \$100M/\$1M per contract = $\boxed{\text{100 contracts long}}$.

(c)
- Futures move: $96.80 - 96.50 = +0.30$ (30 bp). Long P&L $= 100 \times 30 \times \$25 =$ $\boxed{+\$75{,}000}$.
- Interest on deposit: \$100M × 3.20% × 0.25 = $\boxed{\$800{,}000}$.
- Total proceeds: \$800,000 + \$75,000 = $\boxed{\$875{,}000}$.

(d) Implied locked-in rate: $\dfrac{\$875{,}000}{\$100{,}000{,}000 \times 0.25} = 0.0350 = \boxed{3.50\%}$, exactly the day-1 forecast. ✓

Problem 3 — IRS pricing from the yield curve (20 pts)

(a) Discount factors today:

$$e^{-0.025} = 0.9753,\ e^{-0.056} = 0.9456,\ e^{-0.090} = 0.9139,\ e^{-0.124} = 0.8834.$$

$$\sum = 3.7182. \qquad c = \frac{1 - 0.8834}{3.7182} = \frac{0.1166}{3.7182} = \boxed{3.14\%}.$$

(b) One year later, with rates at $\{3.0\%, 3.3\%, 3.5\%\}$:

$$e^{-0.030} = 0.9704,\ e^{-0.066} = 0.9361,\ e^{-0.105} = 0.9003. \qquad \sum = 2.8068.$$

$$V = 100\text{M} \cdot [\,0.0314 \times 2.8068 + 0.9003 - 1\,] = 100\text{M} \cdot (-0.0116) = \boxed{-1.16\text{M}}.$$

(c) Rates have risen, so the new fair fixed coupon (around 3.4%–3.5%) is higher than the locked $c = 3.14\%$. The receiver is stuck collecting a below-market coupon for the remaining 3 years $\Rightarrow$ negative value.

Problem 4 — Currency swap (USD/KRW, 20 pts)

(a) $F_t = 1{,}380 \cdot e^{-0.015t}$:

t (yr)	0.5	1.0	1.5	2.0
F_t (₩/USD)	1,369.7	1,359.5	1,349.3	1,339.2

(b) Method 1 (weighted average of forwards). KRW discount factors at $r = 2.5\%$:

$$e^{-0.0125} = 0.9876,\ e^{-0.0250} = 0.9753,\ e^{-0.0375} = 0.9632,\ e^{-0.0500} = 0.9512;\ \sum = 3.8773.$$

$$K = \frac{0.9876(1{,}369.7) + 0.9753(1{,}359.5) + 0.9632(1{,}349.3) + 0.9512(1{,}339.2)}{3.8773} = \boxed{1{,}354.5 \text{ per USD}}.$$

Method 2 (equivalent): $K = 1{,}380 \cdot (e^{-0.020} + e^{-0.040} + e^{-0.060} + e^{-0.080})/3.8773 = 1{,}380 \cdot 3.8059/3.8773 = 1{,}354.5$. ✓

(c) $K < S_0 = 1{,}380$. USD pays a higher interest rate than KRW ($r_f > r$), so the forward USD price (in KRW) must *decline* over time to prevent arbitrage between holding USD vs. KRW.

(d) The strip of 4 forwards gives *different* KRW payments each period (1,369.7, 1,359.5, 1,349.3, 1,339.2). The swap converts these into a *constant* ₩1,354.5 every period — same PV, smoother cash flow.

Problem 5 — Comparative advantage and the IRS (15 pts)

(a) **Absolute advantage:** AAA in both markets (lower in fixed by 1.50%; lower in floating by 0.70%).
Comparative advantage: AAA in *fixed* (the gap is wider: 1.50% vs. 0.70%). BBB in *floating* (relatively cheaper there).

(b) Total savings $= 1.50\% - 0.70\% = \boxed{0.80\%}$ per year on \$100M (\$800,000 / yr).

(c) Dealer takes 0.10%; remaining 0.70% split equally $\to$ 0.35% each.

- **AAA** target net cost $= (\text{SOFR} + 0.10\%) - 0.35\% = \text{SOFR} - 0.25\%$.
  Cash flows: pays 4% to its bondholders, receives Y from dealer, pays SOFR to dealer.
  Net $= 4\% - Y + \text{SOFR} = \text{SOFR} - 0.25\% \Rightarrow \boxed{Y = 4.25\%}$.
- **BBB** target net cost $= 5.50\% - 0.35\% = 5.15\%$.
  Cash flows: pays SOFR+0.80% to its bondholders, pays X fixed to dealer, receives SOFR from dealer.
  Net $= (\text{SOFR} + 0.80\%) - \text{SOFR} + X = 0.80\% + X = 5.15\% \Rightarrow \boxed{X = 4.35\%}$.
- **Dealer** keeps $X - Y = 4.35\% - 4.25\% = 0.10\%$. ✓

All-in costs: AAA pays SOFR −0.25% (saved 0.35% vs. direct floating); BBB pays 5.15% (saved 0.35% vs. direct fixed); dealer earns 0.10%.

(d) If BBB's credit deteriorates, the next quarterly reset on its floating loan will charge a wider spread, eroding (or eliminating) BBB's savings on the trade.

Problem 6 — Duration-based hedging with futures and swaps (20 pts)

Required reduction. Portfolio dollar duration today: $D_P \cdot V_P = 6.0 \times ₩100\text{B} = ₩600\text{B}$ per 100% yield move. Target: $3.0 \times ₩100\text{B} = ₩300\text{B}$. We must add a derivative position with **negative** dollar duration of $\boxed{-₩300\text{B}}$.

(a) **Alternative 1 — short KTB futures.**
Each contract's dollar duration $= D_F \cdot P_F = 8.0 \times ₩50\text{M} = ₩400\text{M}$.
Number of contracts: $N = \dfrac{₩300\text{B}}{₩400\text{M}} = \boxed{750 \text{ contracts short}}$.

(b) **Alternative 2 — pay-fixed IRS.**
Receive-fixed has $+D_S = +4.3$ years per ₩1 notional; *pay-fixed* has −4.3 years per ₩1.
Required notional: $|N| = \dfrac{₩300\text{B}}{4.3} \approx \boxed{₩69.8\text{B, pay fixed}}$.

(c) Selling KTBs would realize accounting gains/losses, incur bid-ask spreads on a large block, and disrupt the portfolio's carefully-chosen credit and liquidity profile. A derivative overlay adjusts duration *synthetically* with one trade, leaves the cash positions untouched, and unwinds cheaply when the manager wants the original duration back.