BUSS386 Problem Set 5

Swaps

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Problem 1 — Cheapest-to-deliver & futures invoice

A trader has shorted one Treasury bond futures contract. The current settlement price is 92.00 (quoted per $100 face). Three eligible deliverable bonds:

Bond	Quoted price	Conversion factor
A	99.00	1.0500
B	142.00	1.5300
C	119.00	1.2700

Accrued interest on the chosen delivered bond is $2.50 per $100 face value.

(a) Compute the cost of delivering each bond (cost = quoted price − settlement × CF) and identify the cheapest-to-deliver (CTD) bond.

(b) Compute the cash invoice received by the short upon delivering the CTD bond (per $100 face).

(c) In one sentence, why does the CME Treasury bond futures contract need to specify a benchmark coupon (6%) when defining conversion factors?

Problem 2 — SOFR futures hedge

A pension fund expects to deposit $100M for 3 months starting 16 September 2025. Today (1 March 2025), the September 2025 3-month SOFR futures quote is 96.50, implying a SOFR forecast of $100 - 96.50 = 3.50\%$ p.a. Each contract is on $1M notional; one basis point of price change is worth $25 per contract.

(a) Should the pension fund go *long* or *short* SOFR futures to lock in its deposit rate? In one sentence, why?

(b) How many contracts should it trade?

(c) On 16 September 2025, the realized 3-month compounded SOFR is 3.20% (final settlement 96.80). Compute (i) the futures P&L, (ii) the interest earned on the deposit, and (iii) the total proceeds.

(d) Compute the implied locked-in rate (annualized). Does it match the day-1 forecast of 3.50%?

Problem 3 — IRS pricing from the yield curve

Today's KRW zero rates (continuous compounding):

Maturity (yr)	1	2	3	4
Zero rate	2.5%	2.8%	3.0%	3.1%

Consider a 4-year KRW IRS with *annual* payment frequency, notional ₩100M. The fixed-rate payer pays an annual coupon c; the floating-rate payer pays 1-year KOFR (resetting annually).

(a) Compute the fair fixed coupon c using

$$c = \frac{1 - e^{-y_4 \cdot 4}}{e^{-y_1 \cdot 1} + e^{-y_2 \cdot 2} + e^{-y_3 \cdot 3} + e^{-y_4 \cdot 4}}.$$

(b) One year later, the first coupon has been exchanged and the floating leg has just reset. Suppose all KRW zero rates have shifted *up* uniformly by 50 bp (so for the remaining 3 years: 1y = 3.0%, 2y = 3.3%, 3y = 3.5%). Compute the value of the *original* swap, viewed from the *fixed-rate receiver's* perspective, using:

$$V_{\text{fixed-receiver}} = N \cdot \left[c \sum_{i=1}^{3} e^{-y_i \cdot i} + e^{-y_3 \cdot 3} - 1 \right].$$

(c) In one sentence, explain why V in (b) is negative for the fixed-rate receiver.

Problem 4 — Currency swap (USD/KRW)

A Korean importer will pay USD 1M every 6 months for 2 years (4 payments). Today: spot USD/KRW $S_0 = 1{,}380$ per USD; KRW c.c. rate $r = 2.5\%$; USD c.c. rate $r_f = 4.0\%$ (flat curves in both currencies).

(a) Compute the no-arbitrage USD/KRW forward rates F_t at $t \in \{0.5, 1.0, 1.5, 2.0\}$ years.

(b) Compute the constant KRW-per-USD swap rate K that makes a fair currency swap (importer pays K KRW each period and receives 1 USD each period). Use either of:

$$K = \sum_t w_t F_t, \quad w_t = \frac{e^{-rt}}{\sum_s e^{-rs}}; \qquad K = S_0 \cdot \frac{\sum_t e^{-r_f t}}{\sum_t e^{-rt}}.$$

(c) Is K above or below $S_0 = 1{,}380$? In one sentence, why?

(d) In one sentence, what does the swap give the importer that a strip of 4 forwards does not?

Problem 5 — Comparative advantage and the IRS

Two companies want to borrow $100M for 5 years. Quoted rates:

Company	Fixed (5y)	Floating (3M, reset)
AAA Co.	4.0%	SOFR + 0.10%
BBB Co.	5.5%	SOFR + 0.80%

(a) Who has the absolute advantage in each market? Who has the comparative advantage in fixed vs. floating?

(b) Compute the total interest-rate savings if AAA borrows fixed (4%), BBB borrows floating (SOFR + 0.80%), and they enter an IRS to redirect the cash flows.

(c) Suppose a dealer takes 0.10% for arranging the swap, and the remaining savings are split equally between AAA and BBB. Specify the fixed rates exchanged with the dealer so each company saves 0.35%. State each company's all-in net cost.

(d) In one sentence, what would erode the trade's economics if BBB's credit deteriorated mid-life?

Problem 6 — Duration-based hedging with futures and swaps

A Korean pension fund holds a ₩100 billion KRW bond portfolio with modified duration $D_P = 6.0$ years. The fund manager wants to reduce the portfolio duration to $D_P^* = 3.0$ years by adding a *single* derivative position. Consider two alternatives.

Alternative 1 — 10-year KTB futures.

- Modified duration of the CTD bond at delivery: $D_F = 8.0$ years.
- Prepaid forward price: ₩50M per contract.

Alternative 2 — 5-year KRW IRS.

- Effective duration of the receive-fixed leg (vs. floating KOFR): $D_S = 4.3$ years per ₩1 of notional.

(a) For Alternative 1, should the fund go *long* or *short* the futures? Compute the required number of contracts.

(b) For Alternative 2, should the fund *pay* or *receive* fixed? Compute the required notional.

(c) In one sentence, why would a pension fund use derivatives instead of simply selling some of its KTB holdings to reduce duration?