Introduction to Swaps

BUSS386. Futures and Options

Prof. Ji-Woong Chung

Lecture Outline

- Interest Rate Futures
  - Treasury bond futures
  - Eurdollar and SOFR futures
- Swaps
  - Products, pricing and risk management applications
  - Currency, commodity, total rate of return swaps
- Reading: §6.1–6.3 and Ch. 7

Why this lecture matters

- OTC derivatives outstanding (BIS): notional ~\$700T globally. The largest single product family inside that figure is **interest-rate swaps**.
- In Lec02 we built a duration-matched dealer hedge with cash bonds. In practice, virtually *all* of that hedging is done with **IRS** — a stack of forward rate agreements bundled into one contract.
- Today: how the contracts work, how they're priced, the close cousins (currency swaps, commodity swaps, total-return swaps), and the post-LIBOR floating leg (**SOFR**, **KOFR**).

Sources: BIS OTC stats https://www.bis.org/statistics/derstats.htm

Swaps

Swap basics

- A swap is a contract calling for an exchange of payments, on one or more future dates, determined by the difference in two reference prices or interest rates.
- A single-payment swap is equivalent to a cash-settled forward contract.
- A swap provides a means to hedge or speculate on a stream of risky cash flows.
- Traded in over-the-counter market.

Plain-vanilla interest-rate swap — the structure

Notional principal N (**not** exchanged)
Net cash flow on each date $= N \cdot (r_{\text{floating}} - c) \cdot \tau$
where τ is the day-count fraction of the period (e.g. ≈ 0.5 for semi-annual)

- Two parties exchange a stream of fixed coupons for a stream of floating coupons, on the same notional, at regular intervals over the swap's life.
- No principal exchange. Only the *net interest* changes hands.
- An IRS is economically a stack of **forward rate agreements (FRAs)** bundled into one contract.
- Floating leg: today **SOFR** (USD), **KOFR** (KRW), **€STR** (EUR). LIBOR retired end-2021.

Day Counting

Day-count conventions — why we care

A pre-computer-era inheritance: how to count days for accrued interest. Conventions in the form X/Y (X = days in month, Y = days in year):

Convention	Where it's used	Notes
Actual/Actual	US Treasuries, Australia	"True" day count
30/360	US corp & muni bonds, Eurobonds	360-day year, 30-day month
Actual/360	US money market (incl. SOFR)	Inflates rates slightly
Actual/365	**Korea (KTB)**, UK, Japan	365-day year

- In a swap, the fixed and floating legs frequently use *different* conventions — a small but real source of cash-flow mismatch.
- E.g., a USD IRS pays SOFR on Actual/360 but the fixed leg often on 30/360 $\Rightarrow$ the same quoted rate yields slightly different cash flows.
- KRW IRS: fixed leg on Actual/365 (matching KTB convention), floating on KOFR Actual/365 — conventions match domestically.

Reference: https://www.rbcits.com/en/gmi/global-custody/market-profiles.page

Day count in practice — four conventions, one example

4% annual coupon, semi-annual, \$100 face. Partial period March 1 $\rightarrow$ July 3 (124 actual days; 122 days under 30/360; full half-period: 184 actual / 180 under 30/360):

Convention	Calculation	Accrued (per \$100)
Act/Act	$\frac{124}{184} \times \$2$	\$1.348
30/360	$\frac{122}{180} \times \$2$	\$1.356
Act/365	$4\% \times \frac{124}{365} \times \$100$	\$1.359
Act/360	$4\% \times \frac{124}{360} \times \$100$	\$1.378

Useful equivalence

A rate quoted on Act/360 is *not* the same as the same rate on Act/365:
5% Act/360 $= 5\% \times \frac{365}{360} \approx 5.069\%$ Act/365.

Excel: `=DAYS(end, start)` for actual; `=DAYS360(start, end)` for 30/360.

Why we care: day-count basis inside a single IRS

A USD IRS quotes a single fixed rate, but its two legs use *different* day-count conventions. Same 5% rate on \$1M notional, one quarterly period (90 days under 30/360; 92 actual days):

Leg	Quarterly cash flow
Fixed leg, 30/360: \$1M × 5% × 90/360	\$12,500
Floating leg, Act/360: \$1M × 5% × 92/360	\$12,778
Day-count gap	\$278

- Over four quarterly resets, that's roughly \$1,100/year of "free" basis on \$1M notional — not a bug, but priced in by dealers.
- KRW IRS quotes *both* legs on Act/365 (matching KTB convention)

Interest Rate Futures

Treasury futures — specs and quotes

Underlying. A "virtual" bond. The party with the short position can deliver any of a basket of eligible bonds:

- Ultra T-Bond: > 25y   T-Bond: 15–25y   10y note: 6.5–10y
- Benchmark coupon: **6%** (US, since 1999); **5%** for KTB futures (Korea)

Quote conventions.

- US (CME): *thirty-seconds* of a dollar per \$100 face.   e.g., $134'215 = 134 + 21.5/32 = 134.671875$.
- Korea (KRX): decimals.

KRX KTB futures

KRX lists 3-year, 5-year, 10-year, and 30-year KTB futures. All *cash-settled* against a basket of 3 eligible delivery bonds. Notional ₩100M per contract on the 3-year; ₩50M on the 10-year and 30-year. Heavily traded by domestic banks and pension funds for duration management.

Bond quotes — clean vs. dirty price

- Quoted price (a.k.a. **clean price**) is for \$100 face value. e.g., a quote of $90'05$ means $(90 + 5/32) \times 1{,}000 = \$90{,}156.25$ on \$100,000 face.
- The **cash** (or **dirty**) price is what actually changes hands:

$$\text{Cash} = \text{Quoted} + \text{Accrued interest.}$$

Example. March 5, 2013: 11% coupon T-bond maturing July 10, 2028, quoted $95'16 = \$95.50$.

- Last coupon: Jan 10, 2013. Next: Jul 10, 2013.
- Days Jan 10 $\rightarrow$ Mar 5: 54. Days Jan 10 $\rightarrow$ Jul 10: 181.
- Semiannual coupon: $\$100 \times 0.11/2 = \$5.50$.
- Accrued interest: $5.50 \times 54/181 = \$1.64$.
- Cash price (per \$100 face): $95.50 + 1.64 = \boxed{\$97.14}$. On \$100,000 face: \$97,140.

Treasury futures — settlement and the CTD bond

Invoice formula (cash received by the short upon delivery):

$$\text{Cash} = \text{Settlement price} \times \text{Conversion factor (CF)} + \text{Accrued interest.}$$

The CF scales the futures invoice price so different deliverable bonds are comparable.
Cheapest-to-deliver (CTD). The short can pick which bond to deliver. Choose the one minimizing the cost net of the invoice received:

$$\text{Cost of delivery} = \underbrace{\text{Quoted bond price}}_{\text{cost to acquire}} - \underbrace{\text{Settlement price} \times \text{CF}}_{\text{received from invoice}}.$$

Quick example. Settlement 93.25, three candidates:

Bond	Quoted	CF	Cost = Q − Sett.×CF	
1	99.50	1.0382	$99.50 - 93.25(1.0382) = 2.69$	
2	143.50	1.5188	$143.50 - 93.25(1.5188) = \boxed{1.87}$	← CTD
3	119.75	1.2615	$119.75 - 93.25(1.2615) = 2.12$	

Conversion factor — definition, intuition, and why this design

Definition. The CF is the price the deliverable bond would have if priced to yield 6% (the contract's benchmark coupon).

Intuition.

- Coupon $> 6\%$: bond is richer than benchmark $\Rightarrow$ CF $> 1 \Rightarrow$ invoice scaled up.
- Coupon $= 6\%$: matches benchmark $\Rightarrow$ CF $= 1$.
- Coupon $< 6\%$: bond is leaner $\Rightarrow$ CF $< 1 \Rightarrow$ invoice scaled down.

Why this design (basket + CF)?

- *Pinning a single bond* would let it be cornered; shorts couldn't deliver.
- *Basket + CF* lets the short pick the cheapest deliverable, while the CF keeps the playing field fair across coupons. Same trick is used in commodity futures with multiple grades or delivery locations.

Determining the futures price (cost of carry, with coupons)

The exact theoretical price is messy (delivery can be any time in the delivery month, any of several bonds). Assume the CTD and delivery date are known. Then it's just the cost-of-carry on a coupon-paying asset:

$$\boxed{F_0 = (S_0 - I)\, e^{rT},}$$

where S_0 is the cash price of the CTD, I is the PV of coupons paid before delivery, r is the risk-free rate to delivery T.

Determining the futures price — example

Setup: CTD = 12% semi-annual coupon bond, CF = 1.6000; quoted price = \$115; $r = 10\%$ (c.c.); delivery in 270 d. Last coupon 60 d ago; next coupon in 122 d; subsequent in 305 d.

1. **Cash (dirty) price today** (the asset's *actual* price; quoted strips accrued):
$$S_0 = \underbrace{115}_{\text{quoted}} + \underbrace{\tfrac{60}{182}(6)}_{\text{accrued, 60 d into 182-d period}} = \$\mathbf{116.978}.$$
2. **PV of mid-period coupon** (only the +122 d coupon falls inside T; the +305 d coupon does not):
$I = 6\,e^{-0.10\cdot 122/365} = \$\mathbf{5.803}$.
3. **Cost-of-carry to delivery** (own the bond, finance at r, net of the intervening coupon):
$F_0 = (S_0 - I)\,e^{rT} = (116.978 - 5.803)\,e^{0.10\cdot 270/365} = \$\mathbf{119.71}$.
4. **Convert cash → quoted futures** (strip accrued at delivery, divide by CF):
$$Q = \frac{119.71 - \frac{148}{183}(6)}{1.6000} = \mathbf{71.79}.$$

Intuition. Buy the CTD today at S_0, finance at r, collect the +122 d coupon, reinvest it at r to delivery. The net deliverable cost is F_0. We then divide by CF because the futures contract is settled against a synthetic 6% reference bond — the CF rescales the actual CTD to that reference.

SOFR futures (the modern standard)

- The 3-month SOFR futures contract is the dominant short-term interest-rate future in the US (CME). It replaced the Eurodollar future after LIBOR's retirement at end-2021.
- Cash-settled. **Settlement at the end** of the 3-month period to which the rate applies (in arrears, matching how SOFR is referenced in cash markets).
- Final settlement = 100 − compounded daily SOFR over the period. With daily SOFR rates r_i applied to d_i days:

$$R_{\text{period}} = \left[\prod_{i=1}^{n}\left(1 + r_i \cdot \frac{d_i}{360}\right) - 1\right] \cdot \frac{360}{D}, \qquad D = \sum d_i.$$

- One basis-point move in the futures quote = \$25 per contract (\$1M notional, 3-month tenor: \$1M $\times$ 0.0001 $\times$ 0.25).

Legacy: 3-month Eurodollar futures

For decades the world's most-traded interest-rate future (CME). Referenced 3-month USD LIBOR; **settled at the start** of the period (since LIBOR was forward-looking). Last contracts expired April 2023. Same \$25/bp tick.

SOFR futures — example

Q. On May 21, 2025, an investor has agreed to pay *3-month SOFR + 200 bp* on \$100M for the 3-month period beginning Dec 16, 2025. The Dec 2025 SOFR futures quote today is 99.99 (implied SOFR rate = 0.01%). She uses 100 contracts (\$1M each) to lock in her borrowing rate.

Suppose the final settlement on Dec 16, 2025 is 99.20 (compounded SOFR = 0.80%).

A. Futures move: $99.99 - 99.20 = 0.79$ (79 basis points lower).

- She is *short* 100 contracts (gain on rate rise) $\Rightarrow$ gain $= 100 \times \$25 \times 79 = \boxed{\$197{,}500}$.
- Borrowing cost on the loan: $0.80\% + 2.00\% = 2.80\%$ p.a. on \$100M for 3 months = \$700,000.
- Net cost after futures gain: $700{,}000 - 197{,}500 = \boxed{\$502{,}500}$, which is $0.25 \times \$100\text{M} \times 2.01\% \Rightarrow$ effective rate $\approx 2.01\%$ p.a.

She locked in the day-1 SOFR forecast (0.01%) plus the 200 bp spread, with a small basis remaining from the difference between the futures forward and the actual realised compounded SOFR.

(Back to) Swaps

Organization of Trading

- Financial institutions act as market makers and provide bid and ask quotes for the fixed rates that they are prepared to exchange in swaps.

Table 7.4 Example of bid and ask fixed rates in the swap market for a swap where payments are exchanged quarterly (percent per annum).

Maturity (years)	*Bid*	*Ask*	*Swap rate*
2	2.97	3.00	2.985
3	3.05	3.08	3.065
4	3.15	3.19	3.170
5	3.26	3.30	3.280
7	3.40	3.44	3.420
10	3.48	3.52	3.500

- Bid: fixed rate that the market maker pays to receive floating
- Ask: fixed rate that the market maker receive to pay floating.
- Swap rate: the average of the bid and ask rates
- The spread compensates the market maker for its costs.

Interest rate swap pricing

- For floating rate payor, swap is initially equivalent to going long in a fixed rate bond priced at par, and going short in a floating rate bond priced at par
- For the fixed rate payor, the equivalent cash position is the opposite
- In general, we price a swap by finding the difference between the present value of the fixed and floating rate payments.

Valuing the floating leg: par at every reset

Claim. *Just after* each reset, the floating-rate bond is worth F (face value).

Setup. Annual reset; continuous compounding; no credit risk.
At reset t, the rate r_t is observed — it locks the coupon paid at $t+1$, equal to $F(e^{r_t} - 1)$. Principal F repaid at maturity T.

- *Last period.* At $T-1$, only one payment remains: $F\, e^{r_{T-1}}$ at T. Discounting back at r_{T-1} (the rate just set for this period):

$$P_{T-1} \;=\; F\, e^{r_{T-1}} \cdot e^{-r_{T-1}} \;=\; F.$$

- *One step earlier.* At $T-2$, the bond delivers coupon $F(e^{r_{T-2}} - 1)$ at $T-1$ *plus* a bond worth F at $T-1$. Total $= F\, e^{r_{T-2}}$, so $P_{T-2} = F\, e^{r_{T-2}} \cdot e^{-r_{T-2}} = F$.
- By induction, $P_t = F$ at every reset $t = 0, 1, \ldots, T-1$. $\square$

Swap Pricing: A No-Arbitrage Condition

- At swap initiation, the present value of the fixed and floating rate payments must be equal.
  - Because entering into a swap is free, and a voluntary exchange has to be fair to both sides.
- Since the present value of the floating payments equals the face value of the floating rate bond, the present value of the fixed rate payments also must equal the face value of the fixed rate bond.
- Thus, the fixed rate on the swap is determined by setting the present value of the future fixed rate payments equal to par.

Swap Pricing: Implementation

- Imagine that you have derived a spot yield curve $Y_1, Y_2, \ldots, Y_T$ that is appropriate for discounting the fixed rate swap payments

- Then the coupon rate on the swap solves:

$$F = cFe^{-Y_1} + cFe^{-2Y_2} + ... + F(1+c)e^{-TY_T}$$

- Assume that reset frequency is annual.

$$c = \frac{1 - e^{-TY_T}}{e^{-Y_1} + e^{-2Y_2} + ... + e^{-TY_T}}$$

- Note:
  - Swaps are priced to be consistent with the yield curve and hence with implied forward rates, FRAs, and other interest rate forwards and futures
  - Over time the value of the swap changes with market interest rates. Like forward contracts, it is zero sum across the two counterparties.

Example — hedging a bank's balance-sheet risk

Setup. It is December 2025. Southwest savings bank funds \$1M of new *10-year* fixed-rate mortgages (10% p.a.) using rolling *3-month* time deposits (8% p.a. today).

Q. A profit of 2% is locked in for the first 3 months. After that, deposit-rate resets expose the bank to rising rates. Use an interest-rate swap to hedge.

A. Enter a 10-year IRS, semi-annual payments:

- Fixed-rate *payer*: Southwest. Fixed rate $= 8.65\%$.
- Floating leg: 3-month SOFR (compounded in arrears).

After the swap, Southwest's balance sheet on each reset:

$$\underbrace{10\%}_{\text{from mortgages}} - \underbrace{8.65\%}_{\text{paid in swap}} + \underbrace{\text{SOFR}}_{\text{received in swap}} - \underbrace{\text{deposit rate}}_{\text{paid out}} \approx 1.35\% + (\text{spread}),$$

provided 3-month deposit rate $\approx$ SOFR. The mortgage stream now earns a stable spread for the full 10 years.

Example (cont'd) — swap vs. futures, and residual basis

Why a swap beats a strip of futures here.

- One contract covers the entire 10 years.
- Long-dated futures (years 5–10) are illiquid; the swap is a single negotiated price.

But the hedge is not perfect. Residual mismatches remain:

- Mortgages *amortize* — principal balance declines over time, while the swap notional is constant.
- Mortgage payments are *monthly*, not semiannual.
- Mortgages can be *prepaid* (call optionality), shrinking the asset side unpredictably.

$\Rightarrow$ Demand for customized swap structures: **amortizing swaps**, **basis swaps**, **swaptions** (next slide).

Customized Swap Contracts

- Those features of mortgages, which make a plain vanilla swap a less-than perfect hedge for Southwest, is an example of why there is a demand for more specialized swap products such as:
  - Amortizing Swaps
  - Basis Swaps: exchange two floating rates
  - Swaptions: option to enter a swap
- Specialized swaps tend to be more expensive than a plain vanilla swap and a counterparty may be harder to locate

Specialized swaps — examples

1. **Amortizing swap (Southwest, $1 B 10y mortgage book).**
   - Vanilla 10y receive-fixed on $1 B: receives 4.50% on $1 B for all 10 years.
   - Amortizing: notional schedule matches expected mortgage paydown — e.g., $1 B (yr 1) → $850M (yr 3) → $600M (yr 7) → $300M (yr 10).
2. **Basis swap (Korean bank funding mismatch).**
   - KB Kookmin funds 80% in 3M KOFR but lends to corporates referenced to 91-day CD rate.
   - Basis swap: pay 3M KOFR $+x$ bp, receive 91-day CD rate; tenor 3y.
   - Quote: KOFR + 18 bp vs. CD flat. The 18 bp is the **KOFR–CD basis**, reflecting the credit/liquidity gap between secured (KOFR) and unsecured (CD) markets.
3. **Swaption (corporate contingent hedge).**
   - Samsung Electronics plans a $500M bond issue in 6 months, *if* board approves the capex.
   - Risk: rates rise before issue date → funding cost balloons.
   - Buy a 6M × 5Y **payer swaption**, strike 4.75%. Pay 80 bp upfront (≈ $4M).
   - If rates rise to 5.50% and the issue proceeds: exercise, lock 4.75% fixed. If rates fall or the deal is shelved: let it expire.

Why does the IRS market exist?

The comparative-advantage story. A 5-year borrowing example:

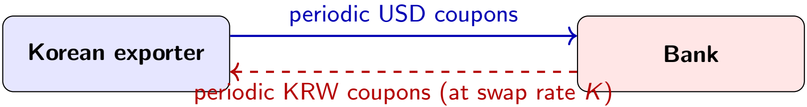
Borrower	Fixed (5y)	Floating (3M, reset)
AAA Corp	4.0%	SOFR + 0.10%
BBB Corp	5.2%	SOFR + 0.60%
Spread (BBB − AAA)	**1.20%**	0.50%

- AAA has the absolute advantage in both, but a *comparatively* larger advantage in fixed (1.20% vs 0.50%).
- AAA borrows fixed at 4.0%, BBB borrows floating at SOFR + 0.60%, then they **swap**: AAA pays SOFR, BBB pays a fixed rate. Total cost saved: **0.70%**, split between the two firms (and a small dealer spread).

Why the spread differential persists (no easy arbitrage)

The 5-year fixed rate is locked. The floating spread is reset every 3 months at the lender's discretion — if BBB's credit deteriorates, the next reset can charge more. So BBB's *expected* long-run floating cost is closer to 5.2% than to today's SOFR+0.60%. The IRS lets each firm exploit its access in the market it can borrow most cheaply.

Currency swap — the structure



Two notionals (in two currencies); often *exchanged* at start and end

- A **currency swap** exchanges interest streams in two different currencies. Variants: fixed–fixed, fixed–floating, floating–floating (basis swap).
- Unlike an IRS, currency swaps usually *do* exchange principals at start *and* maturity.
- Fixed–fixed currency swap is economically a stack of FX forwards bundled into one contract.
- **Korean angle.** The largest single product in the KRW derivatives market is the **USD/KRW cross-currency basis swap (CCS)** — exchanges KRW vs USD floating cash flows. Korean exporters and KTB-issuing utilities use it daily.

Currency swap example — determining the swap rate K

Setup. A US exporter receives €1M every 6 months for 2.5 years.
$S_0 = 1.2673$ \$/€, $r_\$ = 5\%$, $r_e = 3\%$ (flat curves in both).

Forward-rate schedule (from $F_t = S_0\, e^{(r_\$ - r_e)\, t}$):

Maturity (yr)	0.5	1	1.5	2	2.5
F_t (\$/€)	1.2800	1.2929	1.3059	1.3190	1.3323

Swap rate K. Pay €1, receive \$$K$ each period; choose K so PV of swap = 0:

$$K = \sum_t w_t F_t, \qquad w_t = \frac{e^{-r_\$ t}}{\sum_s e^{-r_\$ s}}.$$

Equivalently, using $F_t = S_0\, e^{(r_\$ - r_e) t}$:

$$K = S_0 \cdot \frac{\sum_t e^{-r_e t}}{\sum_t e^{-r_\$ t}} \approx 1.306\,\$/e.$$

Interpretation

Forwards give a *rising* schedule of dollar amounts (\$1.28M → \$1.33M); the swap converts that to a *constant* \$1.306M every period. Same PV, smoother cash flow.

Hedging with swaps versus forwards

- The payoff profile from the sequence of forwards and one swap is different:
  - The sequence of forwards implies the US firm gets less money early on, and more later on (from \$1.28 mil to \$1.3323 mil)
  - The swap implies the firm gets a constant amount \$1.306 mil every payment
- Both strategies perfectly hedge the exposure, as the exchange rate risk is eliminated and both payoff profiles are known at 0. And both have the same present value.
- Differs in liquidity, transaction costs, etc.

Why does currency swap exist?

- The comparative advantage argument makes sense here.

	USD	*AUD*
General Electric	5.0%	7.6%
Qantas Airways	7.0%	8.0%

- Possible sources of comparative advatange
    - Taxes
    - Reputation

Commodity swap — structure and pricing

What it is. Periodic exchange of a fixed price (the swap price) for the floating market price of a commodity. Cash-settled (typical) or physical.

Setup. A company will buy 100,000 barrels of oil in 1 year and again in 2 years. Today's forward prices: $F_1 = \$110$, $F_2 = \$111$. Zero rates: 1y = 6%, 2y = 6.5% (c.c.).

Q. What single fixed swap price x per barrel makes the deal fair?

A. PV of the forward strip (per barrel):

$$V = 110\, e^{-0.06} + 111\, e^{-0.065\cdot 2} = 103.59 + 97.47 = 201.06.$$

Set x so the constant-payment swap has the same PV:

$$x\cdot e^{-0.06} + x\cdot e^{-0.065\cdot 2} = 201.06 \;\Rightarrow\; x\cdot(0.9418 + 0.8781) = 201.06 \;\Rightarrow\; \boxed{x \approx \$110.48}.$$

Commodity swap — what the price reveals

Compare to the forward strip. The swap pays \$110.48 in both years, while forwards would pay \$110 and \$111. So:

- Year 1: the company *overpays* by \$0.48.
- Year 2: the company *underpays* by \$0.52.

The "overpayment in year 1" is implicitly a **loan from the company to the dealer**. The implied 1-year-forward 1-year-out interest rate:

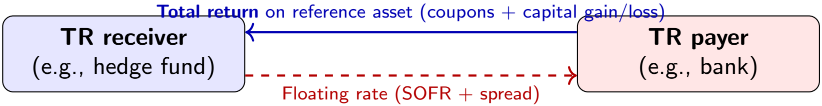
$$0.483 \cdot e^{r_{1,2}} = 0.517 \;\Rightarrow\; r_{1,2} \approx 7.0\%.$$

This matches the implied forward rate from the spot curve:
$r_{1,2} = (0.065 \cdot 2 - 0.06)/(2 - 1) = 7.0\% \checkmark$.

Big picture

A swap is a strip of forwards *plus* an embedded loan that flattens the cash-flow stream. No-arbitrage forces the embedded loan to earn exactly the implied forward rate. Same logic that priced the IRS earlier in this lecture.

Total Return Swap (TRS) — the structure



- One leg: **total return** of a reference asset (bond, loan, equity, index, MBS, EM debt, . . .). Total return = coupons + capital gain/loss.
- Other leg: a **floating rate** (typically SOFR + spread).
- No principal exchange. The TR payer need not own the reference asset; if it does, the TRS hedges its cash flows.

TRS — creating leverage (worked example)

Setup. Three ways to get \$100M of exposure to a reference asset yielding 8.30%. Floating reference: SOFR = 5.80%; HFs face 100 bp spread to SOFR.

	HF A	**HF B**	Cash investor
Asset yield	+8.30%	+8.30%	+8.30%
Pay floating (SOFR)	−5.80%	−5.80%	—
Pay spread to SOFR	−1.00%	−1.00%	—
Net asset spread (on \$100M)	+1.50%	+1.50%	+8.30%
Collateral posted	5%	10%	100%
Implied leverage	20:1	10:1	1:1
Interest on collateral	+5.80%	+5.80%	—
Net return on capital	35.80%	20.80%	8.30%

HF A: $[0.083(100) - 0.068(100) + 0.058(5)]/5 = 35.80\%$. The spread over SOFR compensates the bank for the HF's default risk.

Why TRS exists — both sides have a use

TR receiver (long the asset)

- Off-balance-sheet exposure to a desired asset class.
- Lower administrative cost vs. owning the asset (helpful when the underlying is illiquid or restricted).
- **Leverage** — get $100M of exposure with $5M of collateral (the example above). Main reason hedge funds use TRS.

TR payer (short the asset)

- Hedge price and default risk of an asset they already own — without selling (avoids tax / accounting consequences).
- Get short exposure when direct shorting is restricted (e.g., emerging-market debt).
- Capital relief: many regulatory regimes require less capital against a TRS than the cash position.

In one line

A TRS is a way to slice the *economic exposure* of an asset away from its *legal ownership*. Risk-takers buy exposure cheaply with leverage; risk-shedders keep their position on the books while transferring the price risk.

Duration for forwards, futures and swaps

Why hedge with derivatives instead of cash bonds?

Bridge from Lec 2

The duration-matched hedge in Lec 2 used cash bonds:
\$10M corporate at $D_P = 4.40$ y, hedged by shorting \$5.466M of 10y UST ($D_F = 8.05$).

Today: same hedge, with derivatives.

- **Forward / bond futures.** Replace the short \$5.466M of cash UST with $\sim$50 short futures.
- **IRS.** Replace the entire cash-bond hedge with one swap on the right notional.

Advantages. Lower transaction costs; deeper liquidity at long maturities; no balance-sheet impact; CCP-cleared for swaps (low counterparty risk).

What we need

The dollar duration $D_d \equiv -dV/dy$ of each instrument.
The next slides derive D_d for forwards, futures, and swaps in turn.

Forward on a bond — replication and duration

Replicating portfolio. A forward to buy a bond at delivery date t is:

$$\text{long forward} \equiv \underbrace{\text{long the spot bond}}_{\text{duration } D} - \underbrace{\text{short a } t\text{-year zero}}_{\text{duration } t}$$

E.g.: forward delivery in 3 y on a bond with 1 y remaining at delivery = long a 4y zero − short a 3y zero.

Dollar duration is additive across the portfolio:

$$D_d^{\text{fwd}} = D \cdot S_0 - t \cdot F_0 e^{-rt}$$
$$\overset{(\star)}{=} (D - t) \cdot S_0$$

where $(\star)$ uses the no-arbitrage relation $F_0 e^{-rt} = S_0$ at inception.

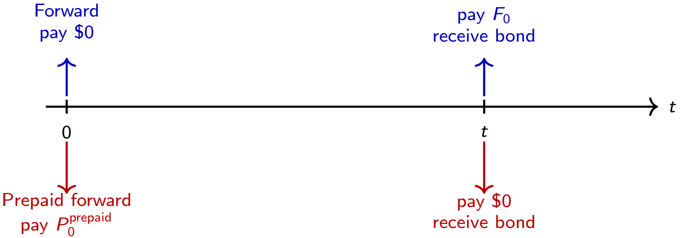
Two equivalent readings

(i) $D_d^{\text{fwd}} = (D - t)\, S_0$: bond duration today, less time-to-delivery, times prepaid forward price.

(ii) $D_d^{\text{fwd}} = D_{\text{Mod}}(\text{bond } \textit{at} \text{ delivery}) \times P^{\text{prepaid}}$: as time passes, the bond's residual life shrinks; the forward inherits the at-delivery duration.

Prepaid forward — the building block

Definition. The *prepaid forward price* P_0^{prepaid} is what you pay *today* for delivery of the asset at future date t, with *no payment at t*.



No-arbitrage link.

$$P_0^{\text{prepaid}} = F_0 \, e^{-rt}$$

- For a *non-dividend / non-coupon* asset: $P_0^{\text{prepaid}} = S_0$.
- For a coupon bond: $P_0^{\text{prepaid}} = S_0 - I < S_0$.

Forward duration — magnitudes and shortcut

The $-t$ correction is not negligible. A bond with $D = 8$ y today:

Time to delivery t	Forward duration $(D - t)$
3 months	7.75 y
3 years	5.00 y
7 years	1.00 y

- For *short* delivery dates, the financing-zero correction is small: $D_d^{\text{fwd}} \approx D_d^{\text{underlying at delivery}}$.
- For *long* delivery dates, ignoring $-t$ materially over-hedges.

Hedging with bond futures — the formula

A bond futures contract is (approximately) a forward on the cheapest-to-deliver (CTD) bond. To hedge a bond portfolio against parallel yield shifts, **equate dollar durations**:

$$D_P \cdot V_P \;=\; N \cdot D_F \cdot V_F$$

$$\Longrightarrow \quad \boxed{N \;=\; \frac{D_P\, V_P}{D_F\, V_F}}$$

- V_P, D_P: market value and modified duration of the portfolio.
- V_F: futures invoice price per contract $=$ quoted price $\times$ contract size.
- D_F: modified duration of the CTD bond.
- Sign: short N contracts to hedge a long bond position; long to hedge a short.

Hedging with bond futures — worked example

Setup. Aug 2, 2025. \$10M bond portfolio; worried about rates rising over the next 3 months. Use Dec 2025 CME T-bond futures.

- Portfolio duration: $D_P = 6.80$ y.
- Futures price: $113'02 = 113.0625 \Rightarrow V_F = \$113{,}062.50$ per contract (\$100k face).
- Duration of CTD: $D_F = 9.20$ y.

$$N = \frac{6.80 \times \$10{,}000{,}000}{9.20 \times \$113{,}062.50} = 65.4 \quad \Rightarrow \quad \boxed{\text{short 65 contracts.}}$$

Swap duration — formula

Setup. A fixed-rate receiver swap is economically:

$$\text{receive-fixed swap} \equiv \text{long fixed-rate bond} - \text{short floating-rate bond}.$$

At inception, $V_{\text{fix}} = V_{\text{flt}} = N$ (notional principal); net swap value zero.

- **Fixed leg.** Modified duration computed in the usual way; call it D_{fix}.
- **Floating leg.** Between resets, prices like a zero of remaining time-to-reset τ_r:

$$D_{\text{eff}}^{\text{flt}} = \frac{\tau_r}{1 + y/k}.$$

At a reset, $\tau_r =$ full period (e.g., 0.25 y for quarterly); approaches 0 just before next reset.

Net swap duration (fixed receiver)

$$D_d^{\text{swap}} = V_{\text{fix}} \cdot D_{\text{fix}} - V_{\text{flt}} \cdot D_{\text{eff}}^{\text{flt}} \approx N \cdot (D_{\text{fix}} - D_{\text{eff}}^{\text{flt}}).$$

The floating-rate receiver's position is the mirror: $-D_d^{\text{swap}}$.

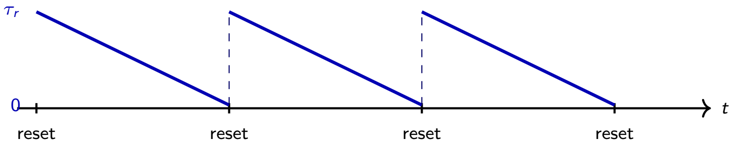
Why the floating leg has tiny (but non-zero) duration

Claim. The effective duration of a floating-rate bond between resets is roughly the time-to-next-reset τ_r, divided by $(1 + y/k)$.

Mechanism

- Just after each reset, the bond is worth par (proved earlier). $P = F$.
- During the period, the next coupon is *locked in* (the rate was set at the reset). The bond temporarily becomes a fixed-coupon zero with τ_r years to its next reset.
- As that "mini-zero," it has duration τ_r. It is *this* small residual sensitivity that the floating leg carries.

Floating-leg duration over time



- "Floating means it tracks rates, so zero rate-sensitivity." True *across* resets.
- *Between* resets, the next coupon is already locked, so the bond is briefly fixed. That locked period is where the duration lives.

Swap duration — worked example

Setup. New 5-year IRS, notional \$1M. Fixed leg: 4.5% s.a. Floating leg: 3-month SOFR (quarterly resets). Current SOFR $\approx$ 4.5%; swap priced at par.

Compute each leg.

- **Fixed bond** (5y, 4.5% s.a., par): $D_{\text{fix}} = 4.43$ y.
- **Floating bond**, just after reset (worst-case max duration): $D^{\text{flt}}_{\text{eff}} = 0.25/(1 + 0.045/4) = 0.247$ y.
- Net duration (fixed receiver): $4.43 - 0.247 = 4.18$ y.

$$D_d^{\text{swap}} \;=\; 4.18 \times \$1{,}000{,}000 \;=\; \boxed{\$41{,}830 \text{ per } 1\% \text{ shift} \;\approx\; \$418/\text{bp}}.$$

Floating receiver: mirror image, $D_d \approx -\$418/\text{bp}$.

LIBOR and Overnight Rates (Optional)

LIBOR vs Overnight reference rates

- LIBOR is being phased out.
- This is tricky for swaps.
  - For example, a 20-year swap negotiated at the end of 2013 will still have 10 years to run as of 2023.
- If banks stop providing LIBOR estimates, it will be necessary for the market to agree on a way of estimating LIBOR from the new reference rates.

Differences between LIBOR and the overnight reference rates

- LIBOR rates are the borrowing rates estimated by banks in the interbank market for periods between one day and one year.
- Overnight rates such as SOFR and SONIA are based on actual transactions between banks.
- The overnight rates are converted to longer reference rates using what might be termed an “averaging process.” Usually the averaging involves daily compounding, but occasionally a simple arithmetic average is used (as for CME’s one-month SOFR futures).
- LIBOR rates for a period are known at the beginning of the period to which they apply, whereas the result of the averaging process for overnight rates is known only at the end of the period.
- LIBOR rates incorporate some credit risk, whereas rates based on overnight rates such as SOFR and SONIA are considered to be risk-free rates.

Overnight Index Swaps

- Swaps based on overnight rates are becoming more popular. These are referred to as overnight indexed swaps (OISs).
- Consider a hypothetical two-year OIS initiated on March 8, 2022, between Apple and Citigroup.
  - Apple agrees to pay to Citigroup interest at the rate of 3% per annum every three months on a notional principal of $100 million, and in return Citgroup agrees to pay Apple the three-month SOFR floating reference rate on the same notional principal.
  - Assume that rates are quoted with quarterly compounding. Ignore the impact of day count conventions and holiday conventions

Overnight Index Swaps

Table 7.1 Cash flows to Apple for one possible outcome of the OIS in Figure 7.1. The swap lasts two years and the notional principal is $100 million.

Date	*SOFR rate (%)*	*Floating cash flow received ($'000s)*	*Fixed cash flow paid ($'000s)*	*Net cash flow ($'000s)*
June 8, 2022	2.20	550	750	−200
Sept. 8, 2022	2.60	650	750	−100
Dec. 8, 2022	2.80	700	750	−50
Mar. 8, 2023	3.10	775	750	+25
June 8, 2023	3.30	825	750	+75
Sept. 8, 2023	3.40	850	750	+100
Dec. 8, 2023	3.60	900	750	+150
Mar. 8, 2024	3.80	950	750	+200

- The difference between LIBOR swap and OIS
  - The LIBOR rate for a period is known at the beginning of the period, whereas the overnight reference rate is not known until the end of the period.
  - The 2.20% floating rate applicable to the first exchange on June 8, 2002, if LIBOR, would be known at the beginning of the swap's life on March 8, 2022

Wrap-up — and a bridge to options

- **Today.** A swap is a stack of forwards bundled into one contract. Same no-arbitrage logic; different cash-flow scheduling.
- Four flavors today:
  - **IRS** — fixed for floating (SOFR / KOFR). The largest derivatives market.
  - **Currency swap** — two interest streams in two currencies; principals usually exchanged.
  - **Commodity swap** — fixed price for floating commodity price.
  - **TRS** — total return on a reference asset for a floating rate.
- **Big picture.** Forwards (Lec 3–4) and swaps (today) are the *linear* derivatives toolkit: payoffs scale 1:1 with the underlying.
- **Lec 6 (next).** *Options* — the first *non-linear* payoff. Right-without-obligation. Asymmetric upside/downside. The single most flexible derivative, and the rest of the course.

Reading for next class: Hull, Ch. 9.