BUSS386 Problem Set 4 — Solutions

Pricing Forwards and Futures

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Problem 1 — Cost-of-carry, non-dividend stock (15 pts)

(a) $F_0 = 80\,e^{0.04} = 80 \cdot 1.0408 = \boxed{\$83.27}$.

(b) Market \$85 > \$83.27 ⇒ forward overpriced. Strategy: short forward, buy stock, borrow \$80.

Action	CF at $t = 0$	CF at $t = 1$
Buy stock at \$80	-80	S_T
Short forward at \$85	0	$85 - S_T$
Borrow \$80 at $r = 4\%$	$+80$	-83.27
Net	0	$\boxed{+\$1.73}$

(c) Market \$82 < \$83.27 ⇒ forward underpriced. Strategy: long forward, short stock, lend proceeds.

Action	CF at $t = 0$	CF at $t = 1$
Short stock (sell now)	$+80$	$-S_T$
Long forward at \$82	0	$S_T - 82$
Lend \$80 at $r = 4\%$	-80	$+83.27$
Net	0	$\boxed{+\$1.27}$

Problem 2 — Discrete dividends + live valuation (20 pts)

(a) PV of dividends today:

$$I = 1\,e^{-0.06 \cdot 2/12} + 1\,e^{-0.06 \cdot 5/12} = 0.99005 + 0.97531 = 1.9654.$$

$$F_0 = (50 - 1.9654)\,e^{0.06 \cdot 0.5} = 48.0346 \cdot 1.03045 = \boxed{\$49.50}.$$

(b) At $t = 3$ months: spot \$48, 3-month maturity remains, one \$1 dividend in 2 months.

$$I' = 1\,e^{-0.06 \cdot 2/12} = 0.9900.$$

$$F_t = (48 - 0.9900)\,e^{0.06 \cdot 0.25} = 47.0099 \cdot 1.01511 = \boxed{\$47.72}.$$

$$f = (F_t - F_0)\,e^{-r(T-t)} = (47.72 - 49.50)\,e^{-0.06 \cdot 0.25} = -1.78 \cdot 0.9851 = \boxed{-\$1.75}.$$

(c) The contract was originally a fair (zero-value) trade, but as the stock fell from \$50 to \$48 the new market forward fell below the locked $F_0 = \$49.50$, so the obligation to buy at \$49.50 is now worse than the prevailing rate ⇒ negative value to the long.

Problem 3 — KOSPI 200 index futures (20 pts)

(a) $F_0 = 405\, e^{(0.025-0.015)\cdot 4/12} = 405\, e^{0.003\overline{3}} = 405 \cdot 1.00334 = \boxed{406.35}$.

(b) Market 410 > fair 406.35 ⇒ futures overpriced. Strategy (per index unit):

Action	CF today	CF at $T = 4\text{m}$
Buy $e^{-q\cdot 4/12} = 0.9950$ basket at 405	-402.99	1 unit basket worth S_T
Short futures at 410	0	$410 - S_T$
Borrow ₩402.99 at $r = 2.5\%$	$+402.99$	$-402.99 \cdot e^{0.025\cdot 4/12} = -406.35$
Net	0	$\boxed{+3.65 \text{ index pts}}$

Buying e^{-qT} baskets and reinvesting dividends gives exactly 1 basket at T (continuous-dividend trick).

(c) Holding the index pays a continuous dividend stream at rate q that partially offsets the cost of borrowing S_0 to buy spot — so the net carry is $r - q$, not r.

Problem 4 — USD/KRW forward (25 pts)

(a) $F_0 = 1{,}380\, e^{(0.025-0.040)\cdot 0.5} = 1{,}380\, e^{-0.0075} = 1{,}380 \cdot 0.99253 = \boxed{1{,}369.7 \text{ per USD}}$.

(b) USD is at a forward *discount* ($F_0 < S_0$). KRW pays less interest than USD, so for there to be no arbitrage between (i) holding USD at r_f and (ii) holding KRW at r then converting at the forward, the forward FX rate must reduce the KRW value of USD.

(c) Market 1,375 > fair 1,369.7 ⇒ forward overpriced. Sell USD forward at the high market price, buy USD spot, finance with KRW borrow:

Action (Korean investor)	CF today (KRW)	CF at $T = 6\text{m}$ (KRW)
Buy $e^{-r_f\cdot 0.5} = 0.9802$ USD spot at ₩1,380	$-1{,}352.71$	USD deposit grows to 1 USD → S_T
Short USD/KRW forward (1 USD at ₩1,375)	0	$1{,}375 - S_T$
Borrow ₩1,352.71 at $r = 2.5\%$	$+1{,}352.71$	$-1{,}352.71 \cdot e^{0.025\cdot 0.5} = -1{,}369.72$
Net	0	$\boxed{+5.28}$

(d) New rates $(r, r_f) = (3.5\%, 4\%)$:

$$F_0' = 1{,}380\, e^{(0.035-0.040)\cdot 0.5} = 1{,}380\, e^{-0.0025} = 1{,}380 \cdot 0.99751 = \boxed{1{,}376.6 \text{ per USD}}.$$

Forward rises from ₩1,369.7 to ₩1,376.6 — the USD discount *narrows*. Higher KRW rate makes holding KRW relatively more attractive, so the USD does not need to be at as steep a forward discount to prevent arbitrage.

(e) **Covered interest parity:** the forward FX premium (or discount) exactly offsets the interest-rate differential between two currencies, so it is impossible to earn a risk-free arbitrage by combining spot FX, the two money markets, and a forward.

Problem 5 — Valuing a live forward (20 pts)

(a) $F_0 = 40\,e^{0.05} = 40 \cdot 1.05127 = \boxed{\$42.05}$.

(b) Three months in, 9 months remain.

$$F_t = 43\,e^{0.05 \cdot 9/12} = 43\,e^{0.0375} = 43 \cdot 1.03821 = \boxed{\$44.64}.$$

$$f_{\text{long}} = (F_t - F_0)\,e^{-r(T-t)} = (44.64 - 42.05)\,e^{-0.0375} = 2.59 \cdot 0.9632 = \boxed{\$2.50}.$$

(c) Cross-check via S_t formula: $f_{\text{long}} = S_t - F_0\,e^{-r(T-t)} = 43 - 42.05 \cdot 0.9632 = 43 - 40.50 = \boxed{\$2.50}$. ✓

(d) With $S_t = \$38$: $F_t = 38 \cdot 1.03821 = \$39.45$.

$$f_{\text{long}} = (39.45 - 42.05)\,e^{-0.0375} = -2.60 \cdot 0.9632 = \boxed{-\$2.50}.$$

Mirror outcome: when the stock falls equally far below its starting point, the long forward loses the same magnitude.