

BUSS386 Problem Set 3 — Solutions

Forwards and Futures

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Note. Problem Set 3 is for *self-study only* — no submission required. These solutions are posted to help you check your work.

Problem 1 — Classification (15 pts)

- (i) **Forward.** Exporter is *short* USD (sells USD, receives KRW). *OTC* — bilateral with the bank, customized notional and date.
- (ii) **Futures.** *Long* (buyer of the index). *Exchange* (KRX) by construction.
- (iii) **Futures.** Farmer is *short* (sells wheat). *Exchange* (CBOT/CME).
- (iv) **Forward.** Jeweller is *long* (buys gold). *OTC* — direct with a dealer, custom size.
- (v) **Futures.** *Short* S&P 500 e-mini to neutralize a long ETF position. *Exchange* (CME).

Problem 2 — Margin accounts and daily settlement (20 pts)

Multiplier $M = 250,000$. Daily P&L = $M \cdot (F_t - F_{t-1})$.

(a) Daily P&L:

- Day 1: $250,000 \times (408 - 405) = +750,000$.
- Day 2: $250,000 \times (402 - 408) = -1,500,000$.
- Day 3: $250,000 \times (393 - 402) = -2,250,000$.
- Day 4: $250,000 \times (397 - 393) = +1,000,000$.

(b) End-of-day balance (before any margin top-up):

Day	0	1	2	3	4
Balance (₩M)	10.00	10.75	9.25	7.00	(→ see (c))

(c) Margin call on **Day 3**: balance ₩7.00M is below maintenance ₩7.50M.

Required deposit to restore initial margin: $10,000,000 - 7,000,000 = 3,000,000$.
After the deposit, Day 3 balance returns to ₩10M. Day 4 close: $10 + 1 = 11$ M.

(d) Cash flows:

- Total cash deposited: 10M (initial) + 3M (call) = 13M.

- Cash returned at close (Day 4 balance): 11M.
- Overall P&L: $11 - 13 = \boxed{-2,000,000}$.

Cross-check: cumulative P&L = $0.75 - 1.50 - 2.25 + 1.00 = -2.00\text{M}$. ✓

Problem 3 — Long hedge with futures (20 pts)

- (a) Korean Air will *buy* fuel \Rightarrow **long** the futures.
 Number of contracts = $200,000/25,000 = \boxed{8}$ contracts at \$90.
- (b) Total cost = (cost at spot) - (long futures gain) = $S_T \cdot Q - (S_T - F_0) \cdot Q = F_0 \cdot Q$.
 With $F_0 \cdot Q = 90 \times 200,000 = \boxed{\$18,000,000}$ for every $S_T \in \{\$84, \$90, \$96\}$.
 Quick check at $S_T = \$84$: $84(200,000) - (84 - 90)(200,000) = 16.8\text{M} + 1.2\text{M} = \18M . ✓
- (c) Algebraically the spot moves cancel: long-futures gain offsets the spot move dollar-for-dollar (perfect hedge assumption).
- (d) Realistic imperfections: (*any one of*) the May futures may not perfectly track the actual jet-fuel grade Korean Air burns; futures expiry may be a few days off; physical jet fuel may be priced at a regional differential to the futures benchmark; or the contract size doesn't divide evenly into the hedge requirement.

Problem 4 — Basis risk on USD/KRW (25 pts)

- (a) Samsung will receive USD and convert to KRW \Rightarrow **short** USD/KRW futures (lock the KRW amount per USD).
 Contracts = $10,000,000/50,000 = \boxed{200}$ short.
- (b) On 31 May:
- Futures P&L (short, gains when futures falls): $(F_{\text{Mar}} - F_{\text{May}}) \cdot \$10\text{M} = (1,390 - 1,370) \cdot 10,000,000 = 200,000,000$ gain.
 - Spot conversion: $1,340 \cdot 10,000,000 = 13,400,000,000$.
 - Total KRW: $13,400,000,000 + 200,000,000 = 13,600,000,000$.
 - Effective rate per USD: $13,600,000,000/10,000,000 = \boxed{1,360}$ per USD.
 Equivalently: $S_{\text{May}} + (F_{\text{Mar}} - F_{\text{May}}) = 1,340 + 20 = 1,360$. ✓
- (c) On 30 June (futures expiry), $F_T = S_T$ exactly (basis = 0). Effective rate = $S_T + (F_{\text{Mar}} - S_T) = F_{\text{Mar}} = \boxed{1,390}$. The hedge perfectly locks the March futures price.
- (d) **Basis** = $S_t - F_t$, the gap between spot and futures. On 31 May the basis was $1,340 - 1,370 = -30$, while at expiry it would have been zero. *Basis risk* is the residual uncertainty that arises when the hedge horizon does not coincide with the futures expiry — the effective rate ends up at F_{Mar} (basis at hedge close), which is not known on March 1.

Problem 5 — Cross hedge / minimum-variance hedge ratio (20 pts)

- (a) $h^* = \rho \cdot \frac{\sigma_S}{\sigma_F} = 0.85 \cdot \frac{0.025}{0.030} = 0.85 \cdot 0.8333 = \boxed{0.7083}$.

- (b) Hedge size = 4,000,000 gallons of jet fuel exposure (long, since the airline will *buy*). To hedge, take **long** heating-oil futures (offsets a fuel-price rise).

$$\text{Contracts} = \frac{h^* \cdot 4,000,000}{42,000} = \frac{2,833,333}{42,000} \approx 67.46 \Rightarrow \boxed{67 \text{ long contracts}}.$$

- (c) Hedge effectiveness = $R^2 = \rho^2 = (0.85)^2 = \boxed{72.25\%}$.
Equivalently, 27.75% of the original variance remains, even with the optimal ratio.
- (d) Heating-oil price changes do not move 1:1 with jet-fuel price changes ($\rho < 1$). The component of jet-fuel price variation that is uncorrelated with heating oil cannot be hedged by any heating-oil position — this is residual basis risk.