Forwards and Futures

BUSS386. Futures and Options

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Lecture Outline

- Definitions
- Payoff of Forward and Futures
  - Daily Settlements
  - Margin Accounts
- Hedging Using Futures
  - Perfect Hedge
  - Imperfect Hedge
- Reading: Ch. 1.3–1.4, Ch. 2 and 3

Forwards & futures — why we care

- A farmer in Iowa, an airline in Hong Kong, and a Korean exporter all face the same problem: a price they need today is set in a market that won't trade until tomorrow.
- **Forwards** (OTC) and **futures** (exchange) are the simplest instrument to lock that future price now.
- Today's anchors:
  - **KOSPI 200 futures** (KRX): Korea's benchmark equity-index future.
  - **S&P 500 e-mini futures** (CME): US benchmark.
  - **USD/KRW forward**: the most-traded Korean OTC derivative.
  - **Brent / WTI oil futures**: the global energy benchmarks.
- Pricing comes next week (Lec 4). Today: *what they are*, *how they pay off*, and *how to hedge with them*.

Futures & Forwards — contract specifications

- **Def.** A futures (or forward) contract is an *agreement* to buy or sell an asset at a future date for a price set today.
- Five things to nail down for any contract:
  - **Underlying asset** (corn, oil, KOSPI 200, USD/KRW, . . .)
  - **Contract size** (e.g. 5,000 bushels of corn; ₩250,000 $\times$ KOSPI 200 index)
  - **Delivery / expiration date**
  - **Forward / futures price** F_0 (set today)
  - **Position** — **long** (buy at T) or **short** (sell at T)
- Entering long or short **costs nothing** upfront (apart from any margin/collateral). Compare with buying a stock: cash leaves your account on day 0.

Forwards vs. Futures — the side-by-side

Forward (OTC)

- Bilateral, customizable size and date
- Settled *once* at expiration
- Counterparty credit risk (governed by ISDA)
- Less liquid, easy to tailor
- E.g. USD/KRW forward at a Korean bank

Futures (exchange)

- Standardized contract, listed on an exchange
- Settled *daily* (mark-to-market) via margin
- CCP becomes counterparty ("novation") $\Rightarrow$ no bilateral credit risk
- Highly liquid, less flexible
- E.g. KOSPI 200 futures on KRX

Same economics, different plumbing

Both lock in the future price. Both have payoff $S_T - F$ at expiry (long position). The differences are in *when cash moves* and *who you face for credit*.

Korean anchor: KOSPI 200 futures vs. S&P 500 e-mini

Spec	**KOSPI 200 futures (KRX)**	**S&P 500 e-mini (CME)**
Multiplier	₩250,000 / index point	$50 / index point
Tick size	0.05 idx pt = ₩12,500	0.25 idx pt = $12.50
Expirations	Quarterly (Mar/Jun/Sep/Dec)	Quarterly (Mar/Jun/Sep/Dec)
Settlement	Cash	Cash
Trading hours	09:00–15:45 KST + night session	Nearly 24h (Sun–Fri)

- In Lec 1, we noted KOSPI 200 *options* were once the world's most-traded by contract count. Futures on the same index sit alongside.
- Both products are *cash-settled* — no one delivers a basket of 200 stocks.

Sources: KRX https://www.krx.co.kr/ CME
https://www.cmegroup.com/markets/equities/sp/e-mini-sandp500.html

Reading a futures quote (CME corn)

CORN FUTURES - CONTRACT SPECS

CONTRACT UNIT	5,000 bushels
PRICE QUOTATION	U.S. cents per bushel
TRADING HOURS	**CME Globex:** Sunday - Friday, 7:00 p.m. - 7:45 a.m. CT and Monday - Friday, 8:30 a.m. - 1:20 p.m. CT TAS: Sunday - Friday 7:00 p.m. - 7:45 a.m. and Monday - Friday 8:30 a.m. - 1:15 p.m. CT **CME ClearPort:** Sunday 5:00 p.m. - Friday 5:45 p.m. CT with no reporting Monday - Thursday from 5:45 p.m. - 6:00 p.m. CT
MINIMUM PRICE FLUCTUATION	1/4 of one cent (0.0025) per bushel = $12.50 TAS: Zero or +/- 4 ticks in the minimum tick increment of the outright
PRODUCT CODE	**CME Globex:** ZC **CME ClearPort:** C **Clearing:** C **TAS:** ZCT
LISTED CONTRACTS	9 monthly contracts of Mar, May, Sep and 8 monthly contracts of Jul and Dec listed annually after the termination of trading in the December contract of the current year.
SETTLEMENT METHOD	Deliverable

Source: CME Group — https://www.cmegroup.com

Hedging with a forward — the farmer's problem

Q. A farmer will harvest 1,000 bushels of corn in 3 months and wants to lock in today's revenue. The 3-month corn forward price is \$3.70/bushel.

Q1. Which forward position locks in the price?

Q2. Three months later, the corn spot price is \$3.62/bushel. What is the payoff on the forward, and what is total revenue?

A.

A1. The farmer will *sell* corn $\Rightarrow$ **short** the forward at $F = \$3.70$ on 1,000 bushels.

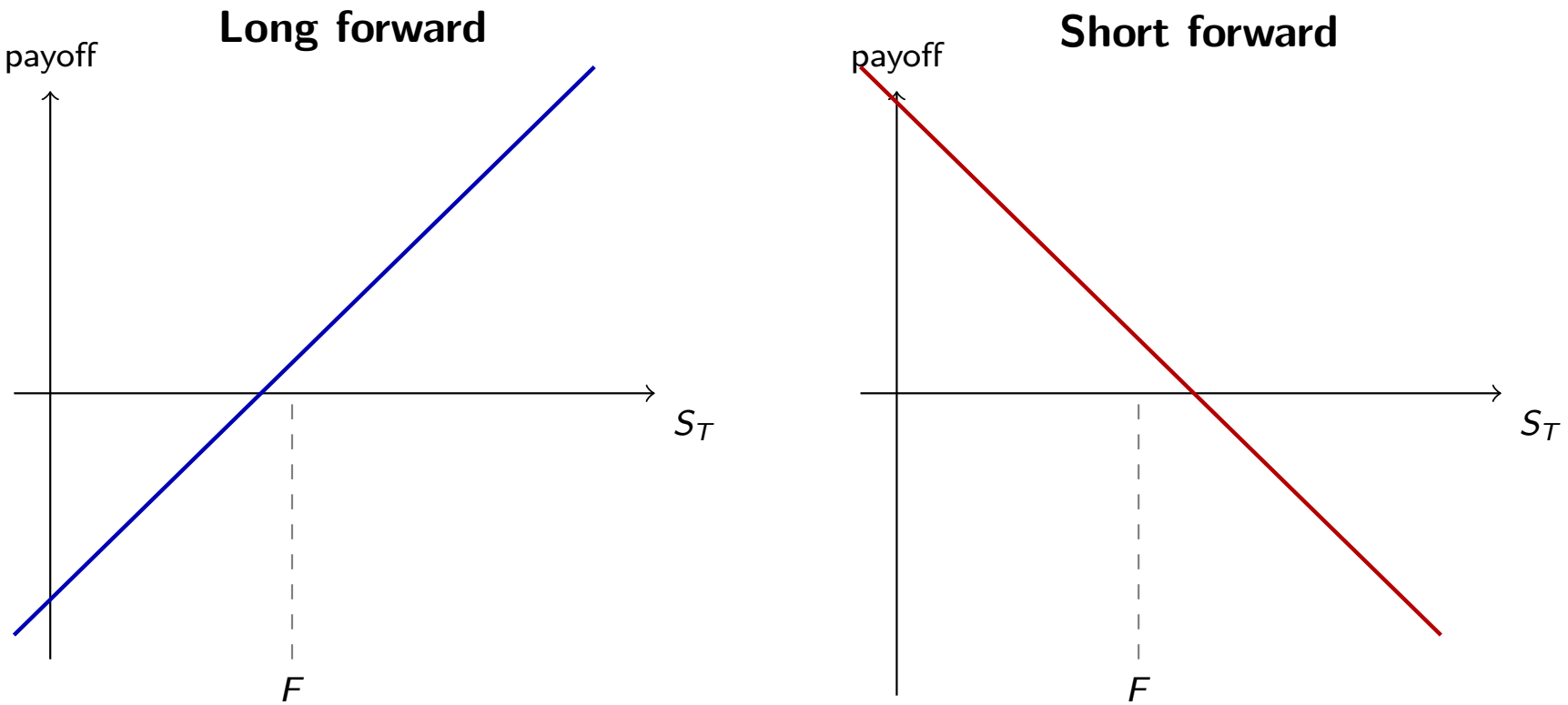
A2. Forward payoff (short) $= F - S_T = 3.70 - 3.62 = +\$0.08$/bushel.
Total revenue = (sell at spot \$3.62) + (forward gain \$0.08) = \$3.70/bushel $\times 1{,}000 = \boxed{\$3{,}700}$.

- Whatever S_T turns out to be, the farmer's revenue is *locked* at $F \cdot Q = \$3{,}700$.

Payoff of a forward — the picture

- Forward is settled once at expiry T. Let F be the forward price set today.
- Payoff at expiry (per unit of underlying):

$$\text{Long: } S_T - F \qquad \text{Short: } F - S_T$$



- **Linear** in S_T, slope ± 1. Crosses zero at $S_T = F$.
- No upfront cost, no upper or lower bound on payoff (in either direction).

Payoff of Futures

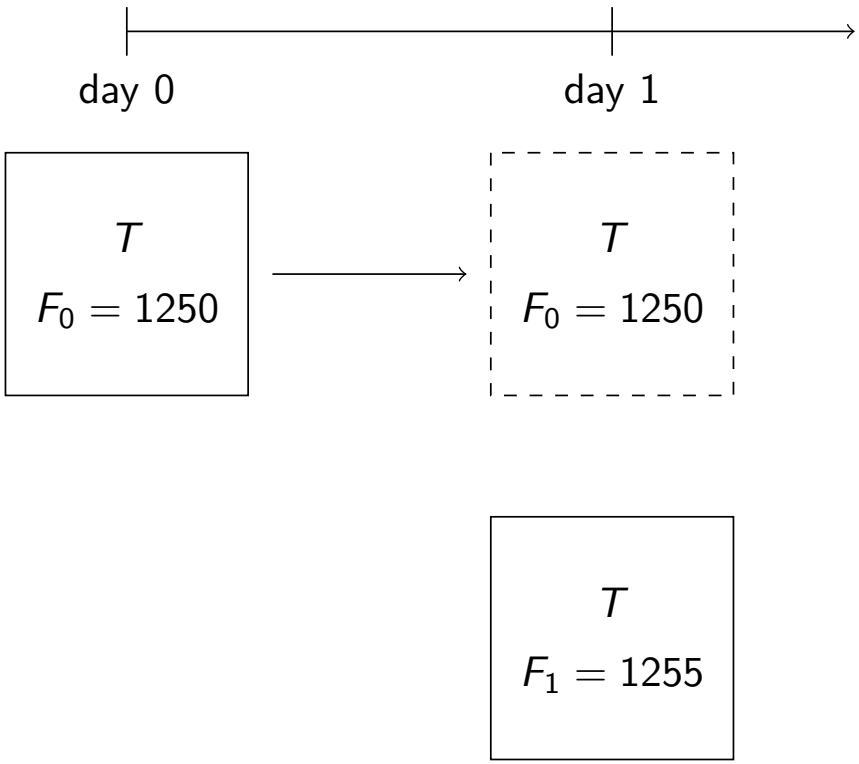
- Suppose we enter a **futures** contract that will expire on date T.
- Unlike forward, the futures will be settled every day.

e.g. Suppose we long futures on gold on day 0 when the futures price is \$1,250 per ounce.

- If later futures prices are as follows, what will be cash flows each day?

Day	Futures price	Daily gain
0	1,250	
1	1,255	?
2	1,248	?
3	1,242	?
⋮	⋮	⋮
T	F_T	?

Payoff of Futures - Closer Look at Day 1



- On day 1, new futures price turns out to be \$1,255.
- The exchange requires the investors from day 0 to abandon the old contract and move to the new one.
- In this settlement, the exchange pays or receives cash to compensate for the price difference.
  - When moving from the old ($F_0 = 1250$) to the new ($F_1 = 1255$), the long-position investor receives \$5.

Payoff of Futures

- On day t, the daily settlement for the long position is as follows.

  1. $$\begin{cases} \text{If } F_t \geq F_{t-1}, & \text{the investor receives } (F_t - F_{t-1}). \\ \text{If } F_t < F_{t-1}, & \text{the investor pays } (F_{t-1} - F_t). \end{cases}$$

     In sum, the daily gain for long position is $F_t - F_{t-1}$.

  2. The investor starts long position in a new contract with F_t.

e.g. Go back to the previous example. Daily gain/loss from the settlement is as follows.

Day	Futures price	Daily gain
0	1,250	
1	1,255	(1,255-1,250) = 5
2	1,248	(1,248-1,255) = -7
3	1,242	(1,242-1,248) = -6
⋮	⋮	⋮
T	F_T	$(F_T - F_{T-1})$

Payoff of Futures - Cumulative Gain

- Suppose we long futures with futures price F_0.
- The futures price on following days turns out to be F_1, F_2, F_3, ..., F_T.
- Assume that the risk-free rate is 0. Then, the cumulative gain from day 1 to day T is

$$\begin{aligned}
&(F_1 - F_0) + (F_2 - F_1) + (F_3 - F_2) + \ldots + (F_T - F_{T-1}) \\
=&F_T - F_0 \\
=&S_T - F_0
\end{aligned}$$

- This is the same as payoff for long position in forward contract.

Operation of Margin Accounts

- Recall that futures contracts are traded on the exchange.
- To prevent investors from defaulting on the contracts, the exchange requires investors to set up a margin account.
- When investors enter a position in futures, they are required to deposit **initial margin** (e.g. \$3,000 per contract).
- Once the margin account is set up, the gain/loss from daily settlement of futures will be added to/subtracted from the account balance.

Operation of Margin Accounts

- As a result of daily settlements, the balance in the margin account changes time to time.

- During the contract period, investors are also required to maintain the balance at a certain level.

  - **Maintenance margin**: the **minimum** amount that must be maintained during the contract.

  - If the balance in the account falls below the maintenance margin, investors receive a **margin call** from exchange. Then, they need to top up the margin account up to the initial margin.

Operation of Margin Accounts - Example

- On day 0, we long a futures contract on gold at the futures price of \$1,250 per ounce. The contract size is 100 ounce per contract.
- Initial margin is \$3,000 and maintenance margin is \$2,000 per contract.

Day	Futures price	Daily gain	Margin account balance	Margin calls
0	1,250		3,000	
1	1,241	(1,241-1,250)$\times$100 = -900	2,100	
2	1,238	(1,238-1,241)$\times$100 = -300	1,800	1,200
3	1,244	(1,244-1,238)$\times$100 = 600	3,600	
4	1,242	(1,242-1,244)$\times$100 = -200	3,400	
$\vdots$	$\vdots$	$\vdots$		

Clearing House

- The **clearing house (CCP)** stands between every buyer and every seller. It nets each member's positions daily.
- Three layers of protection against default:
  1. **Margin from the trader** (initial + maintenance), held at the broker.
  2. **Clearing margin** from each clearing-house member, scaled to the contracts cleared.
  3. **Guaranty fund**: members contribute; tapped if a defaulter's margin runs out.
- Non-members trade through a member, who posts margin on their behalf.

Why it exists

To guarantee that profits get paid — even if the counterparty on the other side of your trade defaults.

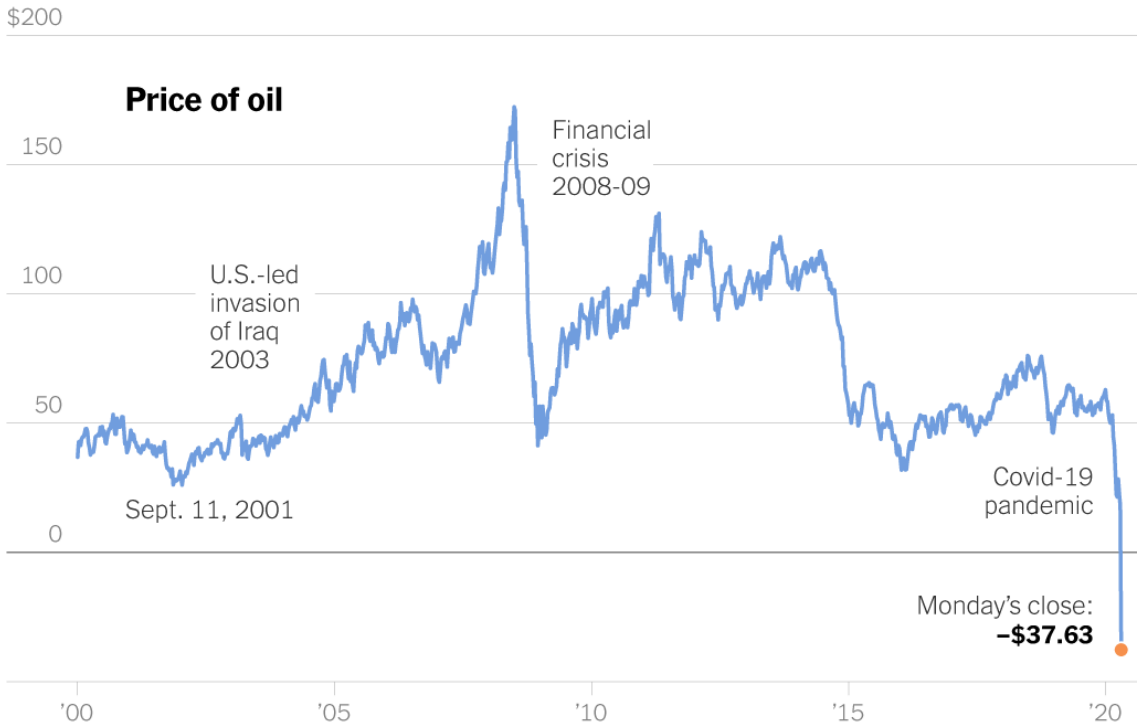
Delivery of Futures

- There are two types of delivery of futures:
  1. Physical delivery: underlying assets are delivered physically (e.g. commodity)
  2. Cash settlements: final daily gain in futures is paid in cash (e.g. stock index)
- Physical delivery may incur additional costs.
  - storage costs
  - transportation costs
  - to feed and look after livestock

Delivery of Futures

- It is possible that the physical delivery costs even outweigh the value of underlying assets.

e.g. On 20 April 2020, the futures price of WTI crude oil becomes **negative**.



[source: https://www.nytimes.com/2020/04/20/business/oil-prices.html]

April 20, 2020 — WTI futures go negative

- The May 2020 WTI futures contract expired April 21. On April 20, it settled at −\$37.63 / **bbl**: long holders *paid* \$37 each to get rid of one barrel of oil.
- Three forces collided:
  1. **Demand collapse.** COVID lockdowns cut global oil consumption ∼25% almost overnight.
  2. **Storage exhausted.** The WTI delivery point is Cushing, Oklahoma; tanks were filling toward capacity.
  3. **Physical delivery obligation.** Anyone still long at expiry had to take delivery — but had nowhere to put the oil and no time to find it.
- Net result: the cost of *accepting delivery* (storage, transport, even waste-disposal fees) exceeded the value of the oil itself.

Brent didn't go negative the same week. Why?

Brent crude (North Sea, traded on ICE) is **cash-settled** against a price index, not physically delivered. No one is forced to take barrels at a fixed location, so the storage-cap mechanism that drove WTI below zero simply does not exist for Brent.

Source: CME Group post-event note — https://www.cmegroup.com/articles/2020/coronavirus-impact-on-may-2020-wti-crude-oil-futures.html

Market Quotes

- Example of futures price quotes

	Open	*High*	*Low*	*Settlement*	*Change*	*Volume*	*Open interest*
Gold 100 oz, $ per oz							
June 2010	1203.80	1216.90	1201.00	1213.40	15.40	194,461	156,156
July 2010	1205.00	1217.50	1202.00	1214.20	15.50	838	714
Aug. 2010	1205.00	1218.70	1202.70	1215.30	15.50	130,676	240,074
Oct. 2010	1208.30	1220.20	1205.30	1217.50	15.60	2,445	21,792
Dec. 2010	1208.80	1222.90	1207.50	1219.90	15.60	7,885	61,497
June 2011	1215.90	1228.00	1215.20	1227.80	15.80	408	13,461

  - Open: the price at which contracts were trading at the beginning of the trading day
  - High: the highest price during the day
  - Low: the lowest price during the day
  - Settlement: the price used for calculating daily gain/loss (usually closing price of the day)
  - (Trading) Volume: the number of contracts traded in a day
  - Open interest: the number of contracts outstanding
    - The total number of (net) long positions or (net) short positions for a certain contract.

Hedging Using Futures

- Hedgers participate in futures market to reduce a particular risk facing them (e.g, fluctuations in oil price, foreign exchange rate).

- To hedge a risk, hedgers take a futures position that neutralizes the risk as much as possible.

  1. Short hedge: a hedge that involves a short position in futures
     - when a hedger expects to **sell** an asset in the future

  2. Long hedge: a hedge that involves a long position in futures
     - when a hedger expects to **buy** an asset in the future

Short hedge — oil producer locking sale price

Q. In May, an oil producer signs a sales contract to deliver *1 million barrels* on 15 August at that day's *spot* price.
August oil futures: $F_0 = \$79/\text{bbl}$. Contract size: 1,000 bbl.

Q1. Position?

Q2. Total revenue on 15 August for $S_T \in \{\$75, \$85\}$?

A.

A1. Seller of oil $\Rightarrow$ **short** $1{,}000{,}000/1{,}000 = \boxed{1{,}000}$ futures contracts at \$79.

A2. Total revenue = (sell at spot) + (futures gain on short, $F_0 - S_T$):

- $S_T = \$75$: $75 \cdot 1\text{M} + (79 - 75) \cdot 1\text{M} = \boxed{\$79\text{M}}$.
- $S_T = \$85$: $85 \cdot 1\text{M} + (79 - 85) \cdot 1\text{M} = \boxed{\$79\text{M}}$.

- Locked revenue = $F_0 \cdot Q = \$79\text{M}$, regardless of S_T.

Long hedge — Korean Air buys jet fuel

Q. In January, Korean Air expects to buy *100,000 barrels* of jet fuel on 15 May. Jet-fuel futures for May delivery: $F_0 = \$92/\text{bbl}$. Contract size: 25,000 bbl.

Q1. Position?

Q2. Total cost on 15 May for $S_T \in \{\$95, \$88\}$?

A.

A1. Buyer of fuel $\Rightarrow$ **long** $100{,}000/25{,}000 = \boxed{4}$ futures contracts at \$92.

A2. Total cost = (buy at spot) − (futures gain):

- $S_T = \$95$: $95 \cdot 100{,}000 - (95 - 92) \cdot 100{,}000 = \boxed{\$9{,}200{,}000}$.
- $S_T = \$88$: $88 \cdot 100{,}000 - (88 - 92) \cdot 100{,}000 = \boxed{\$9{,}200{,}000}$.

- Either way, total cost is locked at $F_0 \cdot Q = \$9.2\text{M}$ — the long futures *offsets* spot moves.

Hedge - General Case

- Suppose that on date 0, we expect to sell asset A on date T.
- To hedge the risk, we short a certain futures contract.
- Total revenue on date T is

$$S_T + (F_0 - F_T)$$

- Depending on how well the futures contract fits the sales plan, the hedge becomes **perfect** or **imperfect**.

Perfect Hedge

- Perfect hedge means eliminating the risk **completely**, thus leaving no risk.

- The hedge using futures becomes perfect when all of the following conditions are satisfied.

  1. The asset whose price is to be hedged is the same as the asset underlying futures contract.

  2. The delivery date of futures contract is the same as the date to buy/sell the underlying asset.

- In this case, the total revenue is

$$S_T + (F_0 - F_T) = \underbrace{(S_T - F_T)}_{=0} + F_0$$

Imperfect Hedge

- Sometimes, we **cannot** find a futures with the perfect match.
- As a second-best way, we try using an alternative contract with the closest delivery month and on the most similar underlying asset.
- The total revenue is

$$S_T + (F_0 - F_T) = \underbrace{(S_T - F_T)}_{\neq 0} + F_0$$

- This does not eliminate the risk completely.

Understanding Basis Risk in Hedging

- The **basis** is the difference between the *spot price* (current market price) and the *futures price* (price agreed for future delivery):

$$\text{Basis} = S_t - F_t$$

- **Basis risk** happens when the spot and futures prices don't move together perfectly, making hedging less effective.
- Think of it like trying to lock in a price for something, but the actual price you get is slightly different because of market changes.

Basis-risk: example

Q. Samsung will receive USD 5,000,000 from a US customer in July. It wants to lock in the KRW value today (March 1).
KRX lists USD/KRW futures for *September only* (delivery dates set quarterly).
Today's quote: $F_{\text{Mar}} = \$1 = 1{,}380$; contract size = USD 50,000.

- Position: **short** 100 contracts ($100 \times \$50{,}000 = \$5\text{M}$).
- In July when USD arrives:
  - Spot $S_{\text{Jul}} = 1{,}340$ per USD (KRW strengthened).
  - September futures $F_{\text{Jul}} = 1{,}355$ per USD.
- Effective rate $= S_{\text{Jul}} + (F_{\text{Mar}} - F_{\text{Jul}}) = 1{,}340 + (1{,}380 - 1{,}355) = \boxed{1{,}365}$ per USD.
- Total KRW $= 1{,}365 \times \$5{,}000{,}000 =$ ₩6.825B.

Why the effective rate $\neq F_{\text{Mar}}$

Samsung receives USD in July but the futures expires in September — so $S_{\text{Jul}} \neq F_{\text{Jul}}$ in general. The leftover gap $(S_{\text{Jul}} - F_{\text{Jul}})$ is the **basis** — it's why this is a hedge, not a perfect lock. If USD were received in September, the basis would be zero ($S_T = F_T$ at expiry) and the rate would lock at F_{Mar} exactly.

Perfect Hedge and Risk Measures

- Suppose that on date 0, an oil company expects to sell gasoline on date T. The gasoline price is normally distributed with the mean of \$50 per barrel and the standard deviation of \$3 per barrel.

Q. Without hedging, what is the bad outcome of the revenue (per barrel) that will happen 5% probability?

Perfect Hedge and Risk Measures

- Go back to the previous question. This time, the company tries to hedge by shorting a gasoline futures with delivery on T. The futures price is \$48 per barrel.

Q1. With the hedging, what is the standard deviation of the revenue?

Q2. With the hedging, what is the bad outcome of the revenue that will happen 5% probability?

Cross Hedge

- **Cross hedge** is a case of imperfect hedge where we hedge the price risk of an asset using futures on a different underlying asset.

e.g. An airline that is concerned about the future price of jet fuel uses futures contract on heating oil.

- Hedge ratio $= \frac{\text{size of underlying assets in futures contract}}{\text{size of exposure}}$
  1. In perfect hedge, hedge ratio $= 1$
  2. In cross hedge, hedge ratio is usually not equal to one. Instead, we choose a particular ratio that will result in the best hedge.

Cross hedge — the minimum-variance hedge ratio

Setup. Hold one unit of asset A (you will sell at T). Short h units of futures on a related but different asset B. Portfolio value change:

$$\Delta V = \underbrace{(S_T - S_0)}_{\Delta S} - h \underbrace{(F_T - F_0)}_{\Delta F}.$$

Variance and minimization.

$$\mathrm{Var}(\Delta V) = \mathrm{Var}(\Delta S) - 2h\,\mathrm{Cov}(\Delta S, \Delta F) + h^2\,\mathrm{Var}(\Delta F).$$

Set $d/dh = 0$:

$$\boxed{h^* = \frac{\mathrm{Cov}(\Delta S, \Delta F)}{\mathrm{Var}(\Delta F)} = \rho\,\frac{\sigma_S}{\sigma_F}.}$$

Two ways to read h^*

(i) Minimum-variance choice of h.
(ii) The OLS slope from regressing ΔS on ΔF. So h^* is exactly the regression beta of the spot change on the futures change — run the regression and read it off.

Cross Hedge - Example

- An airline expects to purchase two million gallons of jet fuel in one month and decides to use heating oil futures for hedging. The standard deviation of futures price is $\sigma_F = 0.0313$, the standard deviation of jut fuel price is $\sigma_S = 0.0263$, and the correlation coefficient is $\rho = 0.928$.

Q1. What is the minimum variance hedge ratio?

Q2. Each of the futures contract is for 42,000 gallons of heating oil. How many contracts does the airline need?

Cross Hedge and Risk Measures

- On date 0, an oil company expects to sell gasoline on date T. The gasoline price is normally distributed with the mean of \$50 and the standard deviation of \$3 per barrel. To hedge the risk, the company wants to use futures. However, the futures on gasoline is not available, so the company instead shorts futures on heating oil with the delivery on date T.
  - The futures price of heating oil is \$75.
  - The minimum variance hedge ratio is found to be 0.24.
  - The heating oil price is normally distributed with the mean of \$80 and the standard deviation of \$10 per barrel.
  - The correlation coefficient between the gasoline and heating oil prices is 0.8.

Q1. What is the distribution of the total revenue (per barrel)?

Q2. With the hedgeing, what is the 5% bad outcome of the total revenue?

Speculation with futures — the leverage story

Why use futures instead of buying the underlying directly?

- **Lower transaction cost:** no need to assemble a basket, no custody, no dividends-in-kind.
- **Leverage:** margin $\ll$ contract notional.
- **Easy short:** no borrow fee, no locate.

Concrete (KOSPI 200 mini)

Index level 400 $\Rightarrow$ contract notional $= 250{,}000 \times 400 = 100\text{M}$.
Initial margin (KRX, $\sim$10%) $\approx 10\text{M}$.

If KOSPI 200 rises 1% (4 pts),

$$\text{P\&L} = 4 \times 250{,}000 = 1{,}000{,}000 \text{ on } ₩10\text{M of margin} = \boxed{+10\%}.$$

Same 1% move on a cash position would give +1%. Futures multiply the gain (and the loss) by $\sim 10\times$.

Source: KRX product spec — https://global.krx.co.kr/

Wrap-up — and a bridge to Lec 4

- **Today.** Forward and futures contracts: specs, daily settlement & margin, payoff, hedging (perfect, imperfect, cross), basis risk, speculation.
- **Big idea.** A forward/future locks in a future price today. At expiry, the long earns $S_T - F$, the short earns $F - S_T$ — linear, no upfront cost, settled bilaterally (forward) or daily through a CCP (future).
- **What we deliberately did not do today.** We never asked *how* F is set. We took the market quote as given.
- **Lec 4 (next week).** Forward/futures *pricing*: the no-arbitrage cost-of-carry formula

$$F_0 = S_0\, e^{(r-q+u)T}$$

linking spot, the risk-free rate, dividends/income q, and storage u. The same formula we just used for hedging will be derived from first principles.

Reading for next class: Hull, Ch. 5.