BUSS386 Problem Set 2

Interest Rates

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Problem 1 — Compounding conversion (15 pts)

A bank quotes you an interest rate of 14% per annum with quarterly compounding.

(a) Find the equivalent rate with continuous compounding.

(b) Find the equivalent rate with annual compounding.

(c) Without computing, predict whether (a) is higher or lower than (b). Briefly justify.

Problem 2 — Zero rates and bond pricing (20 pts)

The 6-month, 12-month, 18-month, and 24-month zero rates (continuously compounded) are:

Maturity	0.5y	1y	1.5y	2y
Zero rate (% c.c.)	4.0	4.5	4.75	5.0

(a) Find the price of a 2-year bond with face value \$100 that pays a 5% per-annum coupon *semiannually* (so \$2.50 every 6 months, plus \$100 face at maturity).

(b) Compute the 2-year *par yield* (continuously compounded) — the coupon rate that makes the bond price equal to its face value.

(c) In one sentence, explain why the par yield in (b) is close to (but not exactly equal to) the 2-year zero rate.

Problem 3 — Forward rates and FRAs (25 pts)

Use the same zero-rate term structure as Problem 2.

(a) Compute the forward rate (continuously compounded) for the period from 18 months to 24 months, $r_0(1.5, 2)$.

(b) Compute the forward rate for the period from 6 months to 12 months, $r_0(0.5, 1)$.

(c) Convert the forward rate from (a) to its semiannually compounded equivalent.

(d) An FRA promises to pay you 7% (semiannually compounded) and receive 6-month SOFR on a notional of \$10,000,000 for the 6-month period starting 18 months from now. Compute the value of the FRA today (use the 24-month zero rate to discount).

(e) In one sentence, what does the sign of your answer in (d) tell you about whether you would prefer to receive fixed or pay fixed at the current forward rate?

Problem 4 — Bond duration (20 pts)

A 5-year bond pays an 8% annual coupon (one payment per year) and has a yield-to-maturity of 11% per annum (continuously compounded). Face value \$100.

(a) Compute the bond's price.

(b) Compute the bond's Macaulay duration (which equals modified duration here, since the yield is continuously compounded).

(c) Use duration to estimate the new price if the yield falls by 0.20 percentage points (from 11.00% to 10.80%).

(d) Recompute the price at $y = 10.80\%$ exactly, and compare with your duration-based estimate from (c). How close are they?

Problem 5 — Duration-matched portfolios and convexity (20 pts)

Two portfolios, both yielding 10% (continuously compounded):

- **Portfolio A (barbell):** a 1-year zero with face \$2,000 plus a 10-year zero with face \$6,000.
- **Portfolio B (bullet):** a 5.95-year zero with face \$5,000.

(a) Compute the value and Macaulay duration of each portfolio. Verify that they have (essentially) the same duration.

(b) For a small parallel yield rise of 0.10% (from 10.00% to 10.10%), use duration to estimate the percentage change in each portfolio's value. Are they (essentially) equal?

(c) For a *large* parallel yield rise of 5% (from 10% to 15%), recompute each portfolio's value *exactly*. Compute the actual percentage change in each. Which portfolio loses less?

(d) In one or two sentences, explain the result in (c) using the concept of *convexity*.

Total: 100 pts.