BUSS386 Problem Set 1 — Solutions

Introduction to Derivatives

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Note. Problem Set 1 is for *self-study only* — no submission required. These solutions are posted to help you check your work.

Problem 1 — Classification (15 pts)

(i) **Forward.** Korean Air is *long*, the fuel supplier is *short*. *OTC* — bespoke size (10M bbl) and counterparty-specific terms; not a standardized exchange product.

(ii) **Forward.** POSCO is *short* (delivers steel, receives fixed price); the auto plant is *long*. *OTC* — physical commercial delivery contract, not a standardized exchange product.

(iii) **Swap** (interest-rate swap). Samsung is the *fixed-rate payer* (long the floating leg); the counterparty is the fixed-rate receiver. *OTC* (vanilla swaps are mandated to clear through a CCP post-2008, but the trade itself is OTC-originated).

(iv) **Futures.** KRX-listed and marked-to-market ⇒ futures. Long = buyer of the contract, short = seller. *Exchange* (KRX, by construction).

(v) **Option** (a call). The buyer is *long* the call; the writer is *short*. *Exchange* — standardized strike (₩200,000) and a 3-month tenor are typical of listed equity options.

Problem 2 — Markets, notional vs. market value (15 pts)

(a) Notional = \$100M. Gross market value = \$1.2M. Market value better reflects current economic exposure: it is what one party would have to pay the other to tear up the contract today; the \$100M never changes hands.

(b) Notional measures the face amount each contract is *written on*; market value measures the present value of the *net* expected cash flows. Most contracts trade close to fair value, so the net expected cash flow is small relative to face. Aggregating millions of such contracts gives a huge gap between notional and market value (about 35:1 in BIS data).

(c)
  (1) **CCP.** Standardized contract specifications and high volumes make central clearing efficient; KRX clears it.

  (2) **Bilateral / ISDA.** Deal-specific (tied to one M&A event), so the contract terms cannot be standardized for a CCP.

  (3) **CCP.** Vanilla swaps between large financial institutions are subject to mandatory central clearing under post-2008 rules (Dodd-Frank in the US, EMIR in the EU).

Problem 3 — Returns and volatility (20 pts)

(a) Simple returns: $R_1 = 2.000\%$, $R_2 = -3.922\%$, $R_3 = 5.102\%$, $R_4 = -1.942\%$.

(b) Log returns: $r_1 = 1.980\%$, $r_2 = -4.001\%$, $r_3 = 4.979\%$, $r_4 = -1.961\%$.

(c) Cumulative Day 0 to Day 4:

- Simple: $\frac{P_4}{P_0} - 1 = \frac{101}{100} - 1 = 1.000\%$ (or, by compounding, $(1+R_1)(1+R_2)(1+R_3)(1+R_4) - 1 = 1.000\%$).
- Log: $r_1 + r_2 + r_3 + r_4 = 0.997\% \approx \ln(101/100) = 0.995\%$.

Log returns are easier over time because they *add*; simple returns must be *compounded*.

(d) Mean of log returns $\bar{r} = 0.249\%$. Sample variance $= \frac{1}{n-1}\sum(r_t - \bar{r})^2 = 0.001610$. Daily $\sigma = \sqrt{0.001610} = 4.013\%$. Annualized $\sigma_{\text{ann}} = 4.013\% \times \sqrt{252} \approx 63.7\%$. (High because $n = 4$ with a large swing; only illustrative.)

(e) The $\sqrt{T}$ rule requires daily returns to be **i.i.d.** (independent and identically distributed). Volatility clustering (autocorrelation in squared returns) violates this in practice.

Problem 4 — Discrete RV (KOSPI 200 scenario) (20 pts)

(a) $E(R) = 0.25(0.22) + 0.55(0.07) + 0.20(-0.18) = 0.0575 = \boxed{5.75\%}$.

(b) $\text{Var}(R) = 0.25(0.1625)^2 + 0.55(0.0125)^2 + 0.20(-0.2375)^2 = 0.017969$.
$\sigma(R) = \sqrt{0.017969} = \boxed{13.40\%}$.

(c) Expected wealth $= 50{,}000{,}000 \times 1.0575 = \boxed{52{,}875{,}000}$.
Worst-case (bear) wealth $= 50{,}000{,}000 \times (1 - 0.18) = \boxed{41{,}000{,}000}$.

(d) $E(R_p) = 0.5(0.0575) + 0.5(0.03) = 0.04375 = \boxed{4.375\%}$.
$\text{Var}(R_p) = (0.5)^2(0.1340)^2 + (0.5)^2(0.06)^2 + 2(0.5)(0.5)(-0.2)(0.1340)(0.06) = 0.004588$.
$\sigma(R_p) = \sqrt{0.004588} = \boxed{6.77\%}$.

(e) Because $\rho = -0.2 < 1$, the two assets do not move in lockstep — the cross-term in the variance formula is *negative* when $\rho < 0$, so combining them lowers risk relative to either one alone.

Problem 5 — VaR and Expected Shortfall (30 pts)

$V = \$2{,}000{,}000$, $\sigma_{\text{daily}} = 1.1\% \Rightarrow$ daily σ in dollars $= \$22{,}000$.

(a) 1-day 95% VaR $= 1.645 \times \$22{,}000 = \boxed{\$36{,}190}$.

(b) 1-day 99% VaR $= 2.326 \times \$22{,}000 = \boxed{\$51{,}172}$.

(c) 10-day 99% VaR $= 2.326 \times \$22{,}000 \times \sqrt{10} = \$51{,}172 \times 3.162 = \boxed{\$161{,}808}$.

(d) Empirical, last 250 days:

(1) 95% VaR $= 0.024 \times \$2{,}000{,}000 = \boxed{\$48{,}000}$.

(2) 95% ES $= 0.030 \times \$2{,}000{,}000 = \boxed{\$60{,}000}$.

Observe that empirical VaR (\$48k) exceeds parametric VaR (\$36k): the recent SPY tail has been fatter than a Gaussian with $\sigma = 1.1\%$ would predict.

(e) (i) ES is the *average* of all returns at or worse than the VaR threshold; since each of those returns is at least as negative as the VaR cutoff, the average is at least as bad as VaR.

(ii) In a long calm period (e.g., a 250-day window with no crash), the empirical tail is shallow, so empirical VaR understates the true tail risk that a future crisis would reveal.

(iii) Examples: exchange margin requirements (KRX/CME initial margin), insurance solvency capital (Solvency II / K-ICS at 99.5% one-year VaR), asset-manager internal risk limits, corporate Cash-Flow-at-Risk for FX/commodity exposure.