BUSS386 Problem Set 1

Introduction to Derivatives

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Note. This problem set is for *self-study only* — you do not need to turn it in. Solutions will be posted on the course website.

Problem 1 — Classification (15 pts)

For each contract below, (a) classify it as a *forward*, *futures*, *swap*, or *option*; (b) state who would naturally be *long* (buyer) and who would be *short* (seller); and (c) say whether it would more naturally trade on an *exchange* or *OTC*, with one short reason.

(i) Korean Air agrees today to buy 10 million barrels of jet fuel at \$80/bbl, with delivery in December 2026.

(ii) POSCO agrees to sell 50,000 tons of steel to a German auto plant in March 2027 at a price fixed today.

(iii) Samsung Electronics agrees to pay 4.0% fixed and receive 3-month SOFR on a notional of \$500M for 5 years.

(iv) A KRX-listed contract obligating the holder to settle KOSPI 200 at 350 in 2 months, marked-to-market daily with initial and variation margin.

(v) A contract giving its holder the right — but not the obligation — to buy 100 shares of NAVER at ₩200,000 per share within the next three months, paid for with a small upfront premium.

Problem 2 — Markets, notional vs. market value (15 pts)

(a) Two banks enter a 5-year interest-rate swap with a notional principal of \$100M. Six months later, the present value of the remaining net payments is \$1.2M (in favor of one side). State the contract's *notional*, *gross market value*, and which number better reflects the true economic exposure. One sentence each.

(b) According to the BIS, OTC derivatives gross notional is roughly \$700T, while gross market value is roughly \$20T. Explain why these two numbers differ by such a large factor.

(c) For each of the following, would the contract more naturally clear through a CCP, or remain bilateral under an ISDA Master Agreement? Justify in one sentence each.

(1) A standardized 3-month KOSPI 200 futures contract.

(2) A bespoke FX option tied to the closing of a specific M&A deal.

(3) A vanilla 5-year USD interest-rate swap between two large G-SIBs.

Problem 3 — Returns and volatility (20 pts)

A stock closes at the following prices on 5 consecutive trading days:

Day	0	1	2	3	4
Close	\$100	\$102	\$98	\$103	\$101

(a) Compute the simple daily returns R_t for $t = 1, \ldots, 4$ (4 numbers).

(b) Compute the log daily returns r_t for $t = 1, \ldots, 4$ (4 numbers).

(c) Compute the cumulative return from Day 0 to Day 4 using both conventions. Which convention is easier to add over time, and why?

(d) Compute the daily standard deviation of the log returns. Annualize it using the $\sqrt{252}$ rule.

(e) In one sentence, state one assumption that is required for the $\sqrt{T}$ scaling rule to hold.

Problem 4 — Discrete random variable (KOSPI 200 scenario) (20 pts)

You believe that next year's return on the KOSPI 200 ETF, R, can be summarized by three scenarios:

Scenario	Return r_i	Probability p_i
Bull market	$+22\%$	0.25
Base case	$+7\%$	0.55
Bear market	-18%	0.20

(a) Compute the expected return $E(R)$.

(b) Compute the variance and the standard deviation $\sigma(R)$.

(c) You invest ₩50,000,000 in this ETF today. What is the expected wealth one year from now? What is the worst-case wealth?

(d) Now consider a second asset, the KTB (Korean Treasury Bond) futures, with $E(R_B) = 3\%$ and $\sigma_B = 6\%$. You hold the ETF and KTB in equal weights ($w_{ETF} = w_{KTB} = 0.5$); the correlation is $\rho = -0.2$. Compute the portfolio's expected return and standard deviation.

(e) In one sentence, explain why the portfolio standard deviation in (d) is lower than the simple weighted average of the two individual standard deviations.

Problem 5 — VaR and Expected Shortfall (30 pts)

You hold a \$2,000,000 position in an S&P 500 ETF. Daily log returns are approximately normal with mean zero and daily standard deviation $\sigma_{\text{daily}} = 1.1\%$.

(a) Compute the parametric (Gaussian) 1-day 95% VaR. Use $z_{0.05} = 1.645$.

(b) Compute the parametric 1-day 99% VaR. Use $z_{0.01} = 2.326$.

(c) Compute the parametric 10-day 99% VaR using the $\sqrt{T}$ scaling rule.

(d) You also pull the last 250 trading days of returns for the same ETF. The 13th-worst return (i.e., the 5th-percentile return) is -2.4%, and the average of the 13 worst is -3.0%.

  (1) Compute the empirical 1-day 95% VaR (in dollars).

  (2) Compute the empirical 1-day 95% Expected Shortfall (in dollars).

(e) In 2–3 sentences each, answer:

  (i) Why does ES exceed VaR by construction?

  (ii) In what kind of market environment would the empirical VaR in (d) likely *understate* true tail risk?

  (iii) Give one real-world application (other than Basel bank capital) where VaR or ES is actually used.

Total: 100 pts.