Introduction to Derivatives

BUSS386. Futures and Options

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Lecture Outline

- Overview of derivatives/markets
- Review: measures of return and risk
- Reading: Hull, Ch. 1.1–1.10 and 22.1–22.3

What are derivatives?

- A derivative is a financial security (i.e., instrument, contract, asset) whose value depends on other underlying **variables**.
- Example: A contract to buy 50,000 barrels of crude oil on September 16, 2017, for $50 per barrel.
- Example: An option contract that gives the holder the right, but not the obligation, to buy 100 shares of a company's stock at $100 per share within the next three months.

What are the underlying variables?

- Usually, the price of a traded assets (e.g, equities, bonds, currencies, commodities)
- or some properties of asset prices (e.g, volatility)
- or some events (e.g., default)
- or weather (e.g. temperature, rainfall), inflation ...

$\Rightarrow$ All variables should be measurable and observable.

Four flavors (we will meet each in turn)

- **Forward** — private agreement to buy/sell X at price K on date T. OTC, no daily settlement.
- **Futures** — standardized forward, exchange-traded, daily margining.
  E.g., CME (Chicago Mercantile Exchange) WTI crude futures; KRX (Korea Exchange) KOSPI 200 futures.
- **Swap** — exchange of two cash-flow streams over time.
  E.g., pay fixed 5%, receive floating SOFR for 5 years. SOFR = Secured Overnight Financing Rate, the USD benchmark that replaced LIBOR in 2023.
- **Option** — the *right, but not the obligation*, to buy (call) or sell (put) at K by/on T. Pay a **premium** upfront for the optionality.

Two families of derivatives

Contract derivatives

- Forwards, futures, swaps, options
- Two counterparties, future cash flows
- Zero-sum: one's gain = other's loss

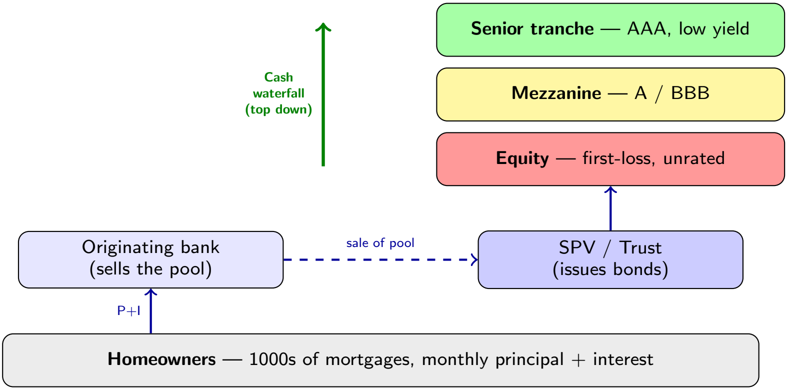
Securitized products

- Pool of underlying assets repackaged into new securities
- E.g., ABS, MBS, CMOs
- Redistributes risk across tranches

Contract derivatives **transfer** risk between two parties.
Securitized products **redistribute** risk that is already in a pool of assets.

This course focuses on *contract* derivatives.

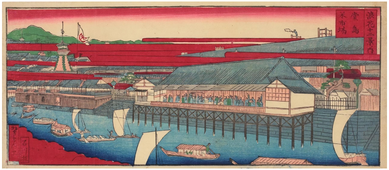
How an MBS is structured



- The bank pools thousands of mortgages and sells them to an **SPV** (Special Purpose Vehicle) — the loans leave the bank's balance sheet.
- The SPV issues bonds, sliced into **tranches** ranked by payment priority:
  - Senior gets paid first, takes losses last ⇒ AAA, low yield.
  - Equity absorbs the first losses ⇒ unrated, high yield.
- A pool of risky mortgages becomes a *layer-cake* of differently-priced bonds, each appealing to a different investor type (pension, insurer, hedge fund).

History of derivatives — pre-modern

- **2000 BC** — India / Arab Gulf trade.
- **300 BC** — Thales (Greece) buys options on olive presses.
- **12th c.** — European merchant fairs use forwards for delivery.
- **1630s** — Amsterdam tulip mania: forwards *and* options on bulbs.
- **1730** — Dojima rice exchange (Osaka): the world's first organized futures market.



Dojima Rice Exchange, Yoshimitsu Sasaki.

History of derivatives — modern

- **1848** — Chicago Board of Trade (**CBOT**): grain futures.
- **1919** — Chicago Mercantile Exchange (**CME**). Later merges with CBOT $\rightarrow$ CME Group.
- **1973** — CBOE lists the first standardized **call options**; puts follow in 1977. Same year: Black–Scholes.
- **1996** — KOSPI 200 futures launch on KRX. Options follow in 1997.

Chicago Board of Trade Building

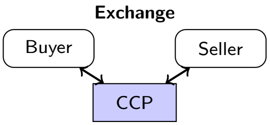
Two habitats: exchange vs. OTC

Exchange-traded

- Standardized contracts
- Central counterparty (**CCP**) clears every trade — you face the exchange, not the original counterparty
- Daily margining, transparent prices
- Futures and listed options
- Examples: CME, CBOE, KRX

Over-the-counter (OTC)

- Bilateral, customizable terms
- Governed by the **ISDA Master Agreement** (netting, collateral, default)
- Counterparty credit risk
- Forwards, swaps, exotics
- Main players: banks, hedge funds, corporates

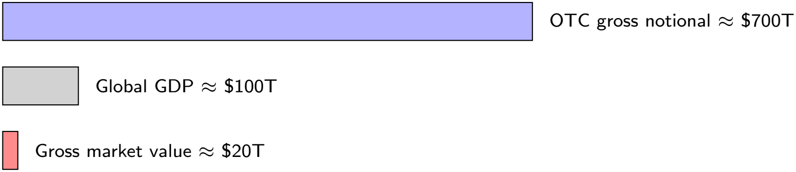


How big is this market? Notional vs. market value

Two different "sizes"

Gross notional = the face amount the contract is written on (e.g., $100M of a 5-year forward). Headline-grabbing, but overstates economic exposure.

Gross market value = what it would cost *today* to replace the contract at current prices. Closer to the real risk number.



Notional overstates risk; market value is the honest number.

Data: BIS https://www.bis.org/statistics/derstats.htm

Korea in one slide

- **KRX** (Korea Exchange) lists: KOSPI 200 index futures/options, KTB (Korean Treasury Bond) futures, USD/KRW futures.
- KOSPI 200 options were the **world's most-traded derivative by contract count** from $\sim$2001 to early 2010s. Retail participation unusually heavy.
- 2011+ tightening reshaped the market via two levers:
  - **Contract multiplier** (KRW per index point) raised $\Rightarrow$ small accounts priced out.
  - **Suitability rules** $\Rightarrow$ brokers must verify knowledge, experience, and capital.

Data: KRX https://data.krx.co.kr/ FSC https://www.fsc.go.kr/

What contributed to rapid growth?

"Necessity is the mother of invention."

- **1971 — Bretton Woods ends.** Currencies float; FX volatility erupts.
  ⇒ **1972** CME launches *currency futures* (the first financial futures).
- **1973 — OPEC oil shock.** Oil triples in months.
  ⇒ Energy hedging demand explodes; oil futures follow in 1983 (NYMEX).
- **1970s — inflation & volatile rates.**
  ⇒ **1975** CBOT launches *interest-rate futures*; T-bond futures 1977.
- **1973 — Black–Scholes published; CBOE opens.** A pricing formula *plus* a venue.
  ⇒ Listed equity options become tradable, not just academic.
- **1980s+ — computing power.** Pricing complex products (swaps, exotics) becomes routine.

Pattern: a shock creates a hedging need → an exchange (or an OTC desk) builds the contract → a model lets people price it.

Four things derivatives are good for

- **Hedging.** Lock in a future price you cannot afford to gamble on.
  Korean Air buys jet-fuel forwards. Samsung sells USD/KRW forwards on export receivables.
- **Speculation with leverage.** Express a view with much less capital than buying the underlying.
  One KOSPI 200 futures contract gives index exposure for a fraction of its notional.
- **Arbitrage.** Lock in profit when two related prices fall out of line.
  If a futures price drifts above its no-arbitrage value, sell the future, buy the basket — almost risk-free P&L.
- **Price discovery.** Futures prices aggregate the market's best guess about the future.
  The Fed and OPEC both watch oil futures curves before acting.

Dangers of derivatives trading

- Three blow-ups, three lessons:
  - **Barings Bank (1995):** unauthorized futures bets sink a 233-year-old bank
    $\Rightarrow$ *instrument was fine; operating model was broken*.
  - **Korean SMEs — KIKO (2008–09):** "cheap" FX options become leveraged short-USD bets when KRW falls
    $\Rightarrow$ *hedge in one tail, leveraged bet in the other*.
  - **UK LDI funds (Sept 2022):** leveraged gilt+swap hedges amplify a gilt sell-off; BoE intervenes
    $\Rightarrow$ *hedge was right; leverage smuggled in liquidity risk*.
- Common thread: either the hedge drifts away from the exposure, or leverage turns a price move into a forced sale.

Refs: BoE FSR Oct 2022
https://www.bankofengland.co.uk/financial-stability-report/2022/october-2022

FSS (KIKO) https://www.fss.or.kr/

The OTC Market Prior to 2008

- The OTC market was largely unregulated.
- Banks acted as market makers, quoting bid and ask prices.
- Transactions between two parties were usually governed by master agreements provided by the International Swaps and Derivatives Association (ISDA https://www.isda.org/).[1]
- Some transactions were cleared through central counterparties (CCPs), which act as intermediaries between the two sides of a transaction, similar to an exchange.

[1] The ISDA is a trade organization of participants in the market for over-the-counter derivatives. ISDA has created a standardized contract (the ISDA Master Agreement) to govern derivative transactions, which helps to reduce legal and credit risks.

Since 2008...

- OTC market has become more regulated. Objectives:
  - Reduce systemic risk
  - Increase transparency
- In the U.S. and other countries, collateral and clearing of trades through a central clearing house (CCP) are required for all standard OTC contracts.
- CCPs must be used to clear standardized transactions between financial institutions in most countries.
- All trades must be reported to a central repository

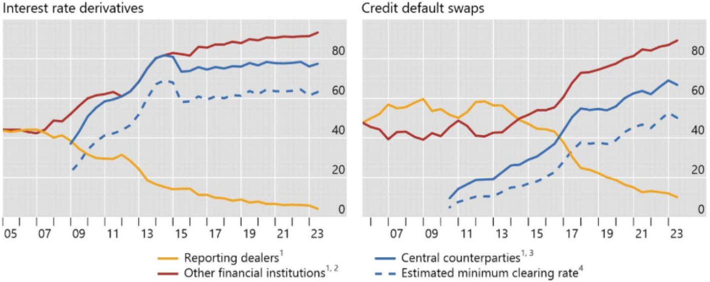
The Lehman Bankruptcy

- Lehman Brothers filed for bankruptcy on September 15, 2008, marking the largest bankruptcy in U.S. history.
- Lehman was heavily involved in the OTC derivatives markets and faced financial difficulties due to high-risk activities and an inability to roll over its short-term funding.
- The firm had hundreds of thousands of outstanding transactions with approximately 8,000 counterparties.
- The process of unwinding these transactions has been challenging for both Lehman's liquidators and their counterparties.

Central clearing: what changed after 2008

Growth of central clearing

Notional amounts outstanding by counterparty, in per cent — Graph A.8



[1] As a percentage of notional amounts outstanding against all counterparties. [2] Including central counterparties but excluding reporting dealers. [3] For interest rate derivatives, data for CCPs prior to end-June 2016 are estimated by indexing the amounts reported at end-June 2016 to the growth since 2008 of notional amounts outstanding cleared through LCH's SwapClear service. [4] Proportion of trades that are cleared, estimated as (CCP / 2) / (1 – (CCP / 2)), where CCP represents the share of notional amounts outstanding that dealers report against CCPs. The CCP share is halved to adjust for the potential double-counting of inter-dealer trades novated to CCPs.

Sources: LCH.Clearnet Group Ltd; BIS OTC derivatives statistics (Table D7 and Table D10.1); BIS calculations.

Source: Bank for International Settlements.

Without a CCP, every OTC trade is a private bilateral exposure — a Lehman-style failure can topple thousands of counterparties in series.
With a CCP, the exchange (or clearing house) sits between every buyer and seller, collects margin daily, and absorbs a default before it spreads.

Statistics: Review

Why we need a probability toolkit

- Every derivative payoff depends on a **future** price we do not yet know.
- To price derivative we need the *distribution* of that future price, not just an average.
- Today's review: random variables $\rightarrow$ Normal distribution $\rightarrow$ three risk measures (SD, VaR, ES).

Hull, Ch. 22.1–22.3.

Simple vs. log returns

- Every P&L, payoff, cash flow ⇒ a return on something. We need a common language.
- **Simple**: $R_t = \dfrac{P_t - P_{t-1}}{P_{t-1}}$ **Log**: $r_t = \ln\left(\dfrac{P_t}{P_{t-1}}\right)$
- Small moves: $r_t \approx R_t$. Big moves diverge: -10% simple $\to$ -10.5% log; -50% simple $\to$ -69% log.

Why log returns in this course

(i) **Additive over time**: two-day log return = day-1 log return + day-2 log return.
(ii) Enables the $\sqrt{T}$ volatility scaling on the next slide.
(iii) Black–Scholes (Lec 10) assumes log-normal prices.

Random variables — discrete (setup)

Example. You buy KOSPI 200 ETF today. What return will you earn over the next year?

Scenario	Return r_i	Probability p_i
Bull market	$+25\%$	0.30
Base case	$+8\%$	0.50
Bear market	-20%	0.20

- Three possible outcomes $\Rightarrow$ a **discrete** random variable R.
- Two questions on the next slide: what return do we *expect*, and how *spread out* are the outcomes?

Random variables — discrete (computation)

- **Expected return** = probability-weighted average:

$$E(R) = \sum_i p_i\, r_i = 0.30(0.25) + 0.50(0.08) + 0.20(-0.20) = \boxed{7.5\%}$$

- **Variance** = probability-weighted squared deviation from $E(R)$:

$$\mathrm{Var}(R) = \sum_i p_i\, (r_i - E(R))^2 = 0.0243$$

- **Standard deviation** (a.k.a. *volatility* in finance):

$$\sigma(R) = \sqrt{\mathrm{Var}(R)} = \boxed{15.6\%}$$

Read this as: "a typical year's return is 7.5%, but it can easily land ±15.6% around that."

Random variables — continuous

- In reality, a stock return is not just three buckets — it can take *any* value on $(-\infty, \infty)$. We call this a **continuous** random variable.
- Instead of a probability table, we get a **probability density function** $f(r)$.
- Probabilities become *areas* under f:

$$\text{Prob}(R \leq 0.05) = \int_{-\infty}^{0.05} f(r)\, dr$$

- Expected return becomes an integral instead of a sum:

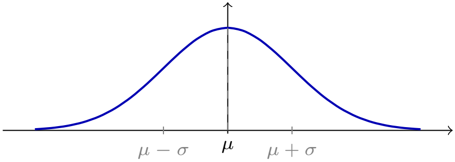
$$E(R) = \int_{-\infty}^{\infty} r\, f(r)\, dr$$

Mental model: the table from the previous slide, but with infinitely many tiny scenarios.

Normal random variables

Density:

$$f(r) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{(r-\mu)^2}{2\sigma^2}}$$



Notation: $R \sim N(\mu, \sigma^2)$.

- Bell-shaped, symmetric around μ.
- σ controls width.
- Linear scaling stays Normal: $aR + b \sim N(a\mu + b,\ a^2\sigma^2)$.

Finance anchor

S&P 500 daily returns: roughly $N(\mu \approx 0.04\%,\ \sigma \approx 1\%)$.
KOSPI 200 daily returns: roughly $N(\mu \approx 0.03\%,\ \sigma \approx 1.2\%)$.

Standard normal — definition and CDF

- A **standard normal** random variable has $\mu = 0$, $\sigma = 1$: $Z \sim N(0,1)$.
- The **cumulative distribution function** $\Phi(x) = \text{Prob}(Z \leq x)$ is the shaded area:

Figure 14.3 Shaded area represents $N(x)$.

- In Excel: `=NORM.S.DIST(x, TRUE)`.

The Z-score: turn any Normal into a standard Normal

- If $R \sim N(\mu, \sigma^2)$, then
$$Z = \frac{R - \mu}{\sigma} \sim N(0, 1).$$
- This is why we only need *one* table (for Φ): subtract μ, divide by σ, look up.
- In Excel, you can skip standardization entirely: `=NORM.DIST(r, mu, sigma, TRUE)`.

Two warm-up exercises:

Ex. 1 $R_1 \sim N(0, 1)$. Find $\text{Prob}(R_1 > 1)$.

Ex. 2 $R_2 \sim N(0.10,\ 0.20^2)$. Find $\text{Prob}(R_2 \leq 0.50)$.

Risk Measures

Risk Measures

- Companies need to assess and manage risks to prevent business failures.
- To have a sense of how risky a project or business is, we can refer to the probability distribution of possible outcomes.
- There are multiple risk measures
  - Standard Deviation
  - Value at Risk (VaR)
  - Expected Shortfalls
  - ...
- Different measures focus on different aspects of the distribution.

Standard Deviation

- Standard deviation measures the level of uncertainty about the outcomes, or the dispersion of probability distribution.

- The larger standard deviation is, the riskier a project.

Ex. Consider the following two projects. Which is riskier?

	Project 1		Project 2	
	good	bad	good	bad
return	10%	0%	0%	-10%
probability	0.5	0.5	0.5	0.5

- A disadvantage of the standard deviation is that it cannot distinguish between upside and downside movement.

Volatility, and the $\sqrt{T}$ scaling rule

- **Volatility** $\sigma =$ standard deviation of returns. The time unit (daily, monthly, annual) matters.
- If daily log returns are i.i.d. with daily standard deviation σ_{daily}:

$$\sigma_{T\text{-day}} = \sigma_{\text{daily}}\sqrt{T} \quad\Longrightarrow\quad \sigma_{\text{annual}} = \sigma_{\text{daily}}\sqrt{252}$$

  (using $\sim$252 trading days per year).
- Rule of thumb: **S&P 500** annualized vol $\approx$ 15–20% in calm years, 30%$+$ in crises. **KOSPI 200** runs slightly higher on average.

Pull data: FRED https://fred.stlouisfed.org/series/SP500 KRX https://data.krx.co.kr/

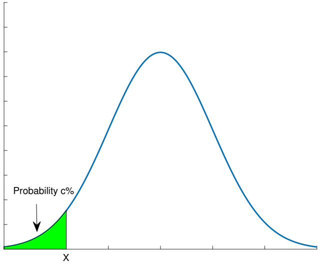
Value at Risk — definition

In one sentence

1-day 95% VaR is the loss level such that, on a typical day, you expect to lose *more* than this only 5% of the time.

Find X such that

$$\text{Prob}(R \leq X) = 0.05$$



- Three knobs: **horizon** (1 day, 10 days), **confidence** (95%, 99%), **portfolio value** V.
- VaR scales with V: doubling the position doubles the VaR.

Read the picture as: "the red 5% tail starts at X. Anything to its left is a worse-than-VaR day."

Value at Risk

- How can we find X satisfying $\Pr(R \leq X) = 0.05$, i.e., 95% VaR?
- In a special case when $R \sim N(\mu, \sigma^2)$, we can find X using the Excel function `norm.inv()`.[2]
  - For given $1 - p$, `norm.inv(1-p,`μ, σ`)` is X that satisfies $\text{Prob}(R \leq X) = 1 - p$.

VaR at 5% = `norm.inv`(0.05,0,1) = -1.645

VaR at 10% = `norm.inv`(0.1,0,1) = -1.282

[2] Closed-form: $VaR(X) = \Phi^{-1}(1 - p)\sigma + \mu$

Value at Risk — example (single stock)

Q. A stock's annual return is $R \sim N(\mu = 15\%, \sigma = 30\%)$. What is the 5% VaR (the threshold the return falls below only 5% of the time)?

A. We need X with $\text{Prob}(R \leq X) = 0.05$.

- Excel: =`NORM.INV(0.05, 0.15, 0.30)` $= -0.343$.
- So $\boxed{X = -34.3\%}$. With 5% probability, the annual return is worse than -34.3%.

Value at Risk — example (S&P 500 ETF)

Q. You hold \$10 M of an S&P 500 ETF. Daily returns: $\mu \approx 0$, $\sigma \approx 1\%$ (so daily σ in dollars $\approx$ \$100,000). What is the 1-day 99% VaR? The 10-day 99% VaR?

Answer. `=NORM.S.INV(0.01)` $= -2.326$. So a normal variable falls by more than 2.326σ with 1% probability.

- 1-day 99% VaR $= 2.326 \times \$100{,}000 = \$232{,}600$.
- 10-day 99% VaR uses the $\sqrt{T}$ rule: $\sigma_{10d} = \sigma_{1d}\sqrt{10}$.

$$\text{10-day VaR} = 2.326 \times \$100{,}000 \times \sqrt{10} = \$735{,}500.$$

Why $\sqrt{T}$? Daily returns are (approximately) independent, so variances add and standard deviations grow with $\sqrt{T}$.

Value at Risk - Multiple Stocks

- Consider a portfolio consisting of n different stocks.
- The return on the portfolio is

$$R_p = \sum_{i=1}^{n} w_i R_i$$

  where w_i is the fraction of wealth invested in stock i.

- If each stock return is normally distributed, then the portfolio return is also normally distributed.

Value at Risk — example (2-stock portfolio)

Q. Portfolio: \$5 M in stock A, \$5 M in stock B (so \$10 M total). Annual returns:

A: $\mu_A = 15\%$, $\sigma_A = 30\%$ B: $\mu_B = 18\%$, $\sigma_B = 45\%$ $\rho_{AB} = 0.4$

Find the 90% VaR.

NB: if $X \sim N(\mu_x, \sigma_x^2)$, $Y \sim N(\mu_y, \sigma_y^2)$, then $X + Y \sim N(\mu_x + \mu_y,\ \sigma_x^2 + \sigma_y^2 + 2\rho\sigma_x\sigma_y)$.

A.

- Portfolio mean: $\mu_p = 0.5(0.15) + 0.5(0.18) = 16.5\%$.
- Portfolio sd:
  $\sigma_p = \sqrt{(0.5 \cdot 0.30)^2 + (0.5 \cdot 0.45)^2 + 2(0.5)(0.5)(0.4)(0.30)(0.45)} = 31.5\%$.
- 90% return threshold:
  $X = \mu_p + \sigma_p \cdot \texttt{NORM.S.INV}(0.10) = 0.165 + 0.315(-1.282) = -23.9\%$.
- In dollars: $\boxed{90\% \text{ VaR} = 0.239 \times \$10{,}000{,}000 = \$2{,}390{,}000}$.

Value at Risk - Historical Data

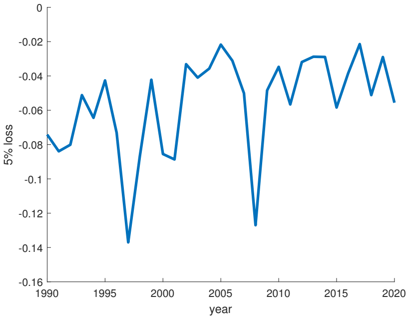
- We can also calculate the VaR using historical data without assuming a specific distribution.
- For example, let's consider 1-year-long historical data of daily returns for a stock price index.
- We aim to estimate the 5% VaR for the next day's return.
- To do this, we assume that the next day's return will be similar to one of the past year's returns.
- The 5% VaR is then the 5th percentile of these historical returns.

Value at Risk - Some Issues I

- VaR estimation is based on the assumption that the distribution of future return is the same as (at least similar to) the distribution of past return.

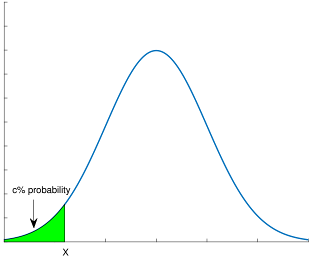
- This assumption may not hold in the real world.

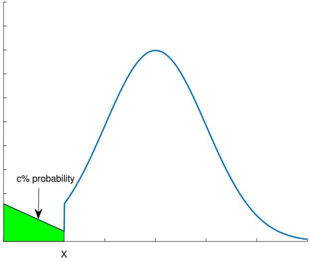
VaR for Index (lowest 5% daily returns)



Value at Risk - Some Issues II

- VaR specifies the **minimum** loss that will occur with a given probability.
- VaR tells nothing about the expected magnitude of the loss.
- Which is the better between the following two?





Expected Shortfall

- Expected Shortfall is another measure to address the shortcoming of VaR.
  - It asks "If things get bad, what is the expected loss?"

- Suppose that we focus on the loss that will happen with 5% probability. Let V denote the 5% loss (VaR). Then, [3]

$$\text{Expected shortfall} = E\left(R \mid R \leq V\right)$$

[3] Under normal distribution: Expected shortfall $= \mu - \sigma \frac{\phi((V-\mu)/\sigma)}{\Phi((V-\mu)/\sigma)}$

Expected Shortfall

- Once historical data are given, we can compute the expected shortfall.
  - In Excel, use "averageif()".

Ex. Let's use the 1-year-long data of daily returns on a stock index.

Q1. What is the expected shortfall with 5% probability?

Q2. What is the expected shortfall with 10% probability?

Where VaR and ES are actually used

- **Bank capital (Basel)** — regulatory capital tied to VaR/ES of the trading book.
  - 1996: capital $= k \times \text{VaR}(99\%, 10\text{d})$, $k \geq 3$.
  - Basel II (2007): VaR-based.
  - Basel III/IV: shifts to **97.5% Expected Shortfall** for trading book.
- **Asset managers & pensions** — internal risk limits; many funds publish daily 95%/99% VaR.
- **Brokerage margin / KRX, CME** — exchanges set initial margin so it covers a 99%+ daily loss (essentially a VaR-style buffer).
- **Insurance (Solvency II, K-ICS)** — 99.5% VaR over a 1-year horizon to size required capital.

Refs: BIS Basel framework https://www.bis.org/basel_framework/ K-ICS (FSS) https://www.fss.or.kr/