Practice Problem: Solution

BUSS386 Futures and Options

1 Implied Volatility

On November 4, 2021, the following options data (379 days until the maturity) was available on the S&P500. Assume that the index is at $4670.00, that the risk-free rate is 0.25% (continuous compounding), and that the dividend yield is 1.17% (continuous compounding). (Base your calculations on the last trade prices of the put or call).

No.	Strike	Call				Put			
		Last	Net	Bid	Ask	Last	Net	Bid	Ask
1	SPX$4600.00	$382.60	+$12.60	$379.5	$382.8	$344.03	+$1.08	$343.3	$346.2
2	SPX$4625.00	$365.98	+$11.88	$363.4	$367.1	$348.29	−$3.66	$352.3	$355.2
3	SPX$4650.00	$321.40	+$00.00	$347.7	$351.4	$387.59	+$0.00	$361.4	$364.4
4	SPX$4675.00	$334.81	+$11.71	$332.3	$336.0	$366.81	−$3.94	$370.9	$373.8
5	SPX$4700.00	$294.80	+$00.00	$317.2	$320.8	$395.38	+$0.00	$380.6	$383.5
6	SPX$4725.00	$266.17	+$00.00	$302.4	$305.9	$425.23	+$0.00	$390.6	$393.5

1. Calculate the implied volatility for each of the call and put options listed. Use Excel's Goal-Seek function using the BSM formula.

2. For the call and put options with a strike price of $4675.00, estimate the option values when volatility decreases to 0.8 times the implied volatility in part (a), and when it increases to 1.2 times the implied volatility in part (a).

Solution:

1.
   - No.1. Call: 19.799649
   - No.2. Call: 19.530899
   - No.3. Call: 17.755202
   - No.4. Call: 19.064806
   - No.5. Call: 17.508827
   - No.6. Call: 16.544449
   - No.1. Put: 19.098960
   - No.2. Put: 18.615259

- No.3. Put: 19.992074
- No.4. Put: 18.143228
- No.5. Put: 18.915052
- No.6. Put: 19.740196

2. - No.4. Call Price with 80% Implied Volatility: 263.324961
   - No.4. Put Price with 80% Implied Volatility: 298.769108
   - No.4. Call Price with 120% Implied Volatility: 406.204730
   - No.4. Put Price with 120% Implied Volatility: 434.781441

2 Implied Volatility

A European call option on a certain stock has a strike price of \$30, a time to maturity of one year, and an implied volatility of 30%. A European put option on the same stock has a strike price of \$30, a time to maturity of one year, and an implied volatility of 33%. What is the arbitrage opportunity open to a trader? Does the arbitrage work only when the lognormal assumption underlying Black-Scholes-Merton holds?

Solution: Put-call parity implies that European put and call options have the same implied volatility. If a call option has an implied volatility of 30% and a put option has an implied volatility of 33%, the call is priced too low relative to the put. The correct trading strategy is to buy the call, sell the put, and short the stock. This does not depend on the lognormal assumption underlying Black-Scholes-Merton. Put-call parity is true for any set of assumptions.

3 Implied Volatility

A stock price is currently \$50 and the risk-free interest rate is 5%. Compute implied volatilities, assuming no dividends. Are the option prices consistent with the assumptions underlying Black-Scholes-Merton?

Stock Price	Maturity = 3 months	Maturity = 6 months	Maturity = 12 months
45	7.00	8.30	10.50
50	3.50	5.20	7.50
55	1.60	2.90	5.10

Solution: Implied volatilities are:

Stock Price	Maturity = 3 months	Maturity = 6 months	Maturity = 12 months
45	37.78	34.99	34.02
50	34.15	32.78	32.03
55	31.98	30.77	30.45

The option prices are not exactly consistent with Black-Scholes-Merton. If they were, the implied volatilities would be all the same. We usually find in practice that low strike price options on a stock have significantly higher implied volatilities than high strike price options on the same stock.

4 Implied Volatilities of Calls and Puts

A European call and put option have the same strike price and time to maturity. The call has an implied volatility of 30% and the put has an implied volatility of 25%. What trades would you do?

Solution: The put has a price that is too low relative to the call's price. The correct trading strategy is to buy the put, buy the stock, and sell the call.

5 Implied Volatilities of Calls and Puts

The market price of a European call is \$3.00 and its price given by Black-Scholes-Merton model with a volatility of 30% is \$3.50. The price given by this Black-Scholes-Merton model for a European put option with the same strike price and time to maturity is \$1.00. What should the market price of the put option be? Explain the reasons for your answer.

Solution: With the notation in the text

$$c_{\text{bs}} + Ke^{-rT} = p_{\text{bs}} + Se^{-qT}$$
$$c_{\text{mkt}} + Ke^{-rT} = p_{\text{mkt}} + Se^{-qT}$$

It follows that

$$c_{\text{bs}} - c_{\text{mkt}} = p_{\text{bs}} - p_{\text{mkt}}$$

In this case, $c_{\text{mkt}} = 3.00$, $c_{\text{bs}} = 3.50$, and $p_{\text{bs}} = 1.00$. It follows that p_{mkt} should be 0.50.

6 OTM Options and Volatility

Option traders sometimes refer to deep-out-of-the-money options as being options on volatility. Why do you think they do this?

Solution: A deep-out-of-the-money option has a low value. Decreases in its volatility reduce its value. However, this reduction is small because the value can never go below zero. Increases in its volatility, on the other hand, can lead to significant percentage increases in the value of the option. The option does, therefore, have some of the same attributes as an option on volatility.

7 Practitioners' Approach

Using the table below, calculate the implied volatility a trader would use for an 8-month option with $K/S_0 = 1.04$.

	K/S_0				
	0.90	0.95	1.00	1.05	1.10
1 month	14.2	13.0	12.0	13.1	14.5
3 month	14.0	13.0	12.0	13.1	14.2
6 month	14.1	13.3	12.5	13.4	14.3
1 year	14.7	14.0	13.5	14.0	14.8
2 year	15.0	14.4	14.0	14.5	15.1
5 year	14.8	14.6	14.4	14.7	15.0

Solution: The implied volatility is 13.45%. We get the same answer by (a) interpolating between strike prices of 1.00 and 1.05 and then between maturities six months and one year and (b) interpolating between maturities of six months and one year and then between strike prices of 1.00 and 1.05.