Practice Problem: Solution

BUSS386 Futures and Options

1 Option Greeks

A financial institution has just sold 1,000 7-month European call options on the Japanese yen. Suppose that the spot exchange rate is 0.80 cent per yen, the exercise price is 0.81 cent per yen, the risk-free interest rate in the United States is 8% per annum, the risk-free interest rate in Japan is 5% per annum, and the volatility of the yen is 15% per annum. Calculate the delta, gamma, vega, theta, and rho of the financial institution's position. Interpret each number.

Solution: $S_0 = 0.80$, $K = 0.81$, $r = 0.08$, $\sigma = 0.15$, $T = 0.5833$, and $q = r_f = 0.05$. Hence, the value of one option is:

$$
\begin{aligned}
\Delta &= e^{-r_f T} N(d_1) = e^{-0.05\times 0.5833} \times 0.5405 = 0.5250 \\
\Gamma &= \frac{N'(d_1)\, e^{-r_f T}}{S_0 \sigma \sqrt{T}} = \frac{0.3969 \times 0.9713}{0.80 \times 0.15 \times \sqrt{0.5833}} = 4.206 \\
\nu &= S_0 \sqrt{T} N'(d_1)\, e^{-r_f T} = 0.80\sqrt{0.5833} \times 0.3969 \times 0.9713 = 0.2355 \\
\Theta &= -\frac{S_0 N'(d_1)\, \sigma e^{-r_f T}}{2\sqrt{T}} + r_f S_0 N(d_1)\, e^{-r_f T} - rKe^{-rT} N(d_2) \\
&= -\frac{(0.8)(0.3969)(0.15)(0.9713)}{2\sqrt{0.5833}} + (0.05)(0.8)(0.5405)(0.9713) - (0.08)(0.81)(0.9544)(0.4948) = -0.0399 \\
\rho &= KTe^{-rT} N(d_2) = (0.81)(0.5833)(0.9544)(0.4948) = 0.2231
\end{aligned}
$$

Delta can be interpreted as meaning that, when the spot price increases by a small amount (measured in cents), the value of an option to buy one yen increases by 0.525 times that amount. Gamma can be interpreted as meaning that, when the spot price increases by a small amount (measured in cents), the delta increases by 4.206 times that amount. Vega can be interpreted as meaning that, when the volatility (measured in decimal form) increases by a small amount, the option's value increases by 0.2355 times that amount. When volatility increases by 1% (= 0.01), the option price increases by 0.002355. Theta can be interpreted as meaning that, when a small amount of time (measured in years) passes, the option's value decreases by 0.0399 times that amount. In particular, when one calendar day passes, it decreases by 0.0399/365 = 0.000109. Finally, rho can be interpreted as meaning that, when the interest rate (measured in decimal form) increases by a small amount, the option's value increases by 0.2231 times that amount. When the interest rate increases by 1% (= 0.01), the option's value increases by 0.002231.

2 Portfolio Insurance

A fund manager has a well-diversified portfolio that mirrors the performance of the S&P 500 and is worth $360 million. The value of the S&P 500 is 1,200, and the portfolio manager would like to buy insurance against a reduction of more than 5% in the value of the portfolio over the next 6 months. The risk-free interest rate is 6% per annum. The dividend yield on both the portfolio and the S&P 500 is 3%, and the volatility of the index is 30% per annum.

1. If the fund manager buys traded European put options, how much would the insurance cost?

2. Create alternative strategies involving traded European call options, and show that they lead to the same result.

Solution:

1. $S_0 = 1200$, $K = 1140$, $r = 0.06$, $\sigma = 0.3$, $T = 0.5$, and $q = 0.03$. Hence, the value of one put option is:

   $$Ke^{-rT}N(-d_2) - Se^{-qT}N(-d_1) = 1140e^{-0.06\times0.5}\times0.4182 - 1200e^{-0.03\times0.5}\times0.3378 = 63.40$$

   The total cost of the insurance is $300,000 \times 63.40 = \$19,020,000$, where $300,000 = \$360\text{m}/1,200$.

2. From the put-call parity:

   $$p = c - S_0e^{-qT} + Ke^{-rT}$$

   This shows that a put option can be created by buying a call option, shorting the index (e^{-qT}), and investing the remainder at the risk-free rate of interest. Applying this to the situation under consideration, the fund manager should:

   (a) Sell $360e^{-0.03\times0.5} = \$354.64$ million worth of shares.

   (b) Buy call options on 300,000 times the S&P 500 with exercise price 1140 and maturity in six months. Each call option is $139.23.

   (c) Invest the remaining ($= \$354.64 - \$41.77 = \$311.87$) cash at the risk-free interest rate of 6% per annum.

   This strategy gives the same result as buying put options directly (Compare payoffs when the index is below K and above K).

3 Risk Management

A financial institution has the following portfolio of over-the-counter options on sterling:

Type	Position	Delta of option	Gamma of option	Vega of option
Call	$-1,000$	0.50	2.2	1.8
Call	-500	0.80	0.6	0.2
Put	$-2,000$	-0.40	1.3	0.7
Call	-500	0.70	1.8	1.4

A traded option is available with a delta of 0.6, a gamma of 1.5, and a vega of 0.8.

1. What position in the traded option and in sterling (the underlying asset) would make the portfolio both gamma neutral and delta neutral?

2. What position in the traded option and in sterling would make the portfolio both vega neutral and delta neutral?

Solution: The delta of the portfolio is

$$-1000 \times 0.50 - 500 \times 0.80 - 2000 \times (-0.40) - 500 \times 0.80 = -450$$

The gamma of the portfolio is

$$-1000 \times 2.2 - 500 \times 0.6 - 2000 \times 1.3 - 500 \times 1.8 = -6000$$

The vega of the portfolio is

$$-1000 \times 1.8 - 500 \times 0.2 - 2000 \times 0.7 - 500 \times 1.4 = -4000$$

1. A long position in 4,000 traded options will give a gamma-neutral portfolio since the long position has a gamma of $4000 \times 1.5 = 6000$. The delta of the whole portfolio (including traded options) is then:

   $$4000 \times 0.6 - 450 = 1950$$

   Hence, in addition to the 4,000 traded options, a short position of 1,950 in sterling is necessary so that the portfolio is both gamma and delta neutral.

2. A long position in 5,000 traded options will give a vega-neutral portfolio since the long position has a vega of $5000 \times 0.8 = 4000$. The delta of the whole portfolio (including traded options) is then

   $$5000 \times 0.6 - 450 = 2550$$

   Hence, in addition to the 5,000 traded options, a short position of 2,550 in sterling is necessary so that the portfolio is both vega and delta neutral.

4 The Black's Model

Consider a futures. Its current price is \$70.00 and its volatility is 16.70% per annum. Suppose the risk-free interest rate is 5.00% per annum (continuous compounding). Use the Black's model to calculate the value of a five-month European put on the futures with a strike price of \$65.00.

Solution: We utilize the Black's model to calculate the option on a futures. In this case, $F_0 = 70$, $\sigma = 16.70\%$, $r = 5\%$, $T = 5/12$, $K = 65$.

$$d_1 = \frac{\ln(F_0/K) + \sigma^2 T/2}{\sigma\sqrt{T}} = 0.74137,$$

and

$$d_2 = \frac{\ln(F_0/K) - \sigma^2 T/2}{\sigma\sqrt{T}} = 0.633572.$$

Hence,

$$p = e^{-rT}\left[KN(-d_2) - F_0 N(-d_1)\right] = 1.038426.$$

5 Delta

What does it mean to assert that the delta of a call option is 0.7? How can a short position in 1,000 options be made delta neutral when the delta of each option is 0.7?

Solution: A delta of 0.7 means that, when the price of the stock increases by a small amount, the price of the option increases by 70% of this amount. Similarly, when the price of the stock decreases by a small amount, the price of the option decreases by 70% of this amount. A short position in 1,000 options has a delta of -700 and can be made delta neutral with the purchase of 700 shares.

6 Delta

Calculate the delta of an at-the-money six-month European call option on a non-dividend-paying stock when the risk-free interest rate is 10% per annum and the stock price volatility is 25% per annum. Use the BSM model.

Solution: In this case, $S_0 = K$, $r = 0.1$, $\sigma = 0.25$, and $T = 0.5$. Also,

$$d_1 = \frac{\ln(S_0/K) + (0.1 + 0.25^2/2)\,0.5}{0.25\sqrt{0.5}} = 0.3712$$

The delta of the option is $N(d_1)$ or 0.64.

7 Portfolio Insurance

Why did portfolio insurance not work well on October 19, 1987?

Solution: Portfolio insurance involves creating a put option synthetically. It assumes that as soon as a portfolio's value declines by a small amount, the portfolio manager's position is rebalanced by either (a) selling part of the portfolio, or (b) selling index futures. On October 19, 1987, the market declined so quickly that the sort of rebalancing anticipated in portfolio insurance schemes could not be accomplished.

8 Delta of Futures Option

What is the delta of a short position in 1,000 European call options on silver futures? The options mature in eight months, and the futures contract underlying the option matures in nine months. The current nine-month futures price is \$8 per ounce, the exercise price of the options is \$8, the risk-free interest rate is 12% per annum, and the volatility of silver futures prices is 18% per annum. Hint: Use Black's model.

Solution: The delta of a European futures call option is usually defined as the rate of change of the option price with respect to the futures price (not the spot price). It is

$$e^{-rT}N\left(d_1\right)$$

In this case, $F_0 = 8$, $K = 8$, $r = 0.12$, $\sigma = 0.18$, $T = 0.6667$

$$d_1 = \frac{\ln(8/8) + (0.18^2/2) \times 0.6667}{0.18\sqrt{0.6667}} = 0.0735$$

$N\left(d_1\right) = 0.5293$ and the delta of the option is

$$e^{-0.12 \cdot 0.6667} \times 0.5293 = 0.4886$$

The delta of a short position in 1,000 futures options is therefore -488.6. Hence, a long position in nine-month futures on 488.6 ounces is necessary to hedge the option position.

Extra: If you want to hedge the option position using the underlying asset (i.e., silver), how many units of silver do you need? (Assume no storage cost for silver)
The delta of a nine-month futures contract is $e^{0.12 \cdot 0.75} = 1.094$ assuming no storage costs. (This is because silver can be treated in the same way as a non-dividend-paying stock when there are no storage costs. $F_0 = S_0 e^{rT}$). Hence, the delta of the option with respect to silver price change is (the delta of the option with respect to futures price change) times (the delta of the futures with respect to silver price changes), i.e., $\frac{\mathrm{dc}}{\mathrm{d}P} = \frac{\mathrm{dc}}{\mathrm{d}F} \cdot \frac{\mathrm{d}F}{\mathrm{d}P}$. Therefore, we need a long position in $-488.6 \times 1.094 = -534.6$ ounces of silver.

9 Delta and Gamma

A company uses delta hedging to hedge a portfolio of long positions in put and call options on a currency. Which of the following would give the most favorable result?

1. A virtually constant spot rate
2. Wild movements in the spot rate

Explain your answer.

Solution: A long position in either a put or a call option has a positive gamma. When gamma is positive, the hedger gains from a large change in the stock price and loses from a small change in the stock price. Hence, the hedger will fare better in case (b).

10 Gamma and Vega

Under what circumstances is it possible to make a European option on a stock index both gamma neutral and vega neutral by adding a position in one other European option with the same underlying asset but different K and T? Use the BSM model.

Solution: Assume that S_0, K, r, σ, T, q are the parameters for the option held and $S_0, K^*, r, \sigma, T^*, q$ are the parameters for another option. Suppose that d_1 has its usual meaning and is calculated on the basis of the first set of parameters while d_1^* is the value of d_1 calculated on the basis of the second set of parameters. Suppose further that w of the second option are held for each of the first option held. The gamma of the portfolio is:

$$\alpha\left[\frac{N'(d_1)\,e^{-qT}}{S_0\sigma\sqrt{T}} + w\frac{N'(d_1^*)\,e^{-qT^*}}{S_0\sigma\sqrt{T^*}}\right]$$

where α is the number of the first option held.
Since we require gamma to be zero:

$$w = -\frac{N'(d_1)\,e^{-q(T-T^*)}}{N'(d_1^*)}\sqrt{\frac{T^*}{T}}$$

The vega of the portfolio is:

$$\alpha\left[S_0\sqrt{T}N'(d_1)\,e^{-qT} + wS_0\sqrt{T^*}N'(d_1^*)\,e^{-qT^*}\right]$$

Since we require vega to be zero:

$$w = -\sqrt{\frac{T}{T^*}}\frac{N'(d_1)\,e^{-q(T-T^*)}}{N'(d_1^*)}$$

Equating the two expressions for w

$$T^* = T$$

Hence, the maturity of the option held must equal the maturity of the option used for hedging.

11 The Put-Call Parity and Greeks

Use the put-call parity relationship to derive, for a non-dividend-paying stock, the relationship between:

1. The delta of a European call and the delta of a European put.
2. The gamma of a European call and the gamma of a European put.
3. The vega of a European call and the vega of a European put.
4. The theta of a European call and the theta of a European put.

Solution:

1. For a non-dividend paying stock, put-call parity gives at a general time t:

   $$p + S = c + Ke^{-r(T-t)}$$

   Differentiating with respect to S:

   $$\frac{\partial p}{\partial S} + 1 = \frac{\partial c}{\partial S}$$

   or

   $$\frac{\partial p}{\partial S} = \frac{\partial c}{\partial S} - 1$$

   This shows that the delta of a European put equals the delta of the corresponding European call less 1.0.

2. Differentiating with respect to S again

   $$\frac{\partial^2 p}{\partial S^2} = \frac{\partial^2 c}{\partial S^2}$$

   Hence, the gamma of a European put equals the gamma of a European call.

3. Differentiating the put-call parity relationship with respect to σ

   $$\frac{\partial p}{\partial \sigma} = \frac{\partial c}{\partial \sigma}$$

   showing that the vega of a European put equals the vega of a European call.

4. Differentiating the put-call parity relationship with respect to t

   $$\frac{\partial p}{\partial t} = rKe^{-r(T-t)} + \frac{\partial c}{\partial t}$$

   This is in agreement with the thetas of European calls and puts given in the lecture note since $N(d_2) = 1 - N(-d_2)$.

12 Hedging using Greeks

A financial institution has the following portfolio of over-the-counter options on sterling:

Type	Position	Delta of Option	Gamma of Option	Vega of Option
Call	$-1,000$	0.5	2.2	1.8
Call	-500	0.8	0.6	0.2
Put	$-2,000$	-0.40	1.3	0.7
Call	-500	0.70	1.8	1.4

A traded option is available with a delta of 0.6, a gamma of 1.5, and a vega of 0.8.

1. What position in the traded option and in sterling would make the portfolio both gamma neutral and delta neutral?
2. What position in the traded option and in sterling would make the portfolio both vega neutral and delta neutral?

Solution: The delta of the portfolio is

$$-1,000 \times 0.50 - 500 \times 0.80 - 2,000 \times (-0.40) - 500 \times 0.70 = -450$$

The gamma of the portfolio is

$$-1,000 \times 2.2 - 500 \times 0.6 - 2,000 \times 1.3 - 500 \times 1.8 = -6,000$$

The vega of the portfolio is

$$-1,000 \times 1.8 - 500 \times 0.2 - 2,000 \times 0.7 - 500 \times 1.4 = -4,000$$

1. A long position in 4,000 traded options will give a gamma-neutral portfolio since the long position has a gamma of $4,000 \times 1.5 = +6,000$. The delta of the whole portfolio (including traded options) is then:

   $$4,000 \times 0.6 - 450 = 1,950$$

   Hence, in addition to the 4,000 traded options, a short position of 1,950 in sterling is necessary so that the portfolio is both gamma and delta neutral.

2. A long position in 5,000 traded options will give a vega-neutral portfolio since the long position has a vega of $5,000 \times 0.8 = +4,000$. The delta of the whole portfolio (including traded options) is then

   $$5,000 \times 0.6 - 450 = 2,550$$

   Hence, in addition to the 5,000 traded options, a short position of 2,550 in sterling is necessary so that the portfolio is both vega and delta neutral.

13 Portfolio Insurance

Suppose that \$70 billion of equity assets are the subject of portfolio insurance schemes. Assume that the schemes are designed to provide insurance against the value of the assets declining by more than 5% within one year. Suppose $S_0 = 70$, $K = 66.5$, $T = 1$. Other parameters are estimated as $r = 0.06$, $\sigma = 0.25$, and $q = 0.03$. Calculate the value of the stock or futures contracts that the administrators of the portfolio insurance schemes will attempt to sell if the market falls by 23% in a single day.

Solution: We can regard the position of all portfolio insurers taken together as a single put option. Compute the value of the option:

$$\begin{aligned} d_1 &= \frac{\ln(70/66.5) + (0.06 - 0.03 + 0.25^2/2)}{0.25} = 0.4502 \\ N(d_1) &= 0.6737 \end{aligned}$$

The delta of the option is

$$\begin{aligned} & e^{-qT}[N(d_1) - 1] \\ &= e^{-0.03}(0.6737 - 1) \\ &= -0.3167 \end{aligned}$$

This shows that 31.67% or \$22.17 billion of assets should have been sold before the decline. After the decline, $S_0 = 53.9$, $K = 66.5$, $T = 1$, $r = 0.06$, $\sigma = 0.25$, and $q = 0.03$.

$$\begin{aligned} d_1 &= \frac{\ln(53.9/66.5) + (0.06 - 0.03 + 0.25^2/2)}{0.25} = -0.5953 \\ N(d_1) &= 0.2758 \end{aligned}$$

The delta of the option has dropped to

$$e^{-0.03 \cdot 1}(0.2758 - 1) = -0.7028$$

This shows that cumulatively 70.28% of the assets originally held should be sold. An additional 38.61% of the original portfolio should be sold. The sales measured at pre-crash prices are about \$27.0 billion. At post-crash prices, they are about \$20.8 billion.

14 Hedging with Delta and Gamma

A bank's position in options on the dollar-euro exchange rate has a delta of 30,000 and a gamma of $-80,000$. Explain how these numbers can be interpreted. The exchange rate (dollars per euro) is 0.90. What position would you take to make the position delta neutral? After a short period of time, the exchange rate moves to 0.93. Estimate the new delta. What additional trade is necessary to keep the position delta neutral? Assuming the bank did set up a delta-neutral position originally, has it gained or lost money from the exchange-rate movement?

Solution: The delta indicates that when the value of the euro exchange rate increases by \$0.01, the value of the bank's position increases by $0.01 \times 30,000 = \$300$. The gamma indicates that when the euro exchange rate increases by \$0.01, the delta of the portfolio decreases by $0.01 \times 80,000 = 800$. For delta neutrality, 30,000 euros should be shorted. When the exchange rate moves up to 0.93, we expect the delta of the portfolio to decrease by $(0.93 - 0.90) \times 80,000 = 2,400$, so that it becomes 27,600. To maintain delta neutrality, it is therefore necessary for the bank to unwind its short position by 2,400 euros so that a net 27,600 have been shorted. When a portfolio is delta neutral and has a negative gamma, a loss is experienced when there is a large movement in the underlying asset price. (The short position experienced a loss since the price has gone up.) We can conclude that the bank is likely to have lost money.

15 Option Greeks and Binomial Tree

According to the Black-Scholes-Merton (BSM) model, the delta of a European option on a non-dividend-paying stock has the analytical expressions. We can also compute the delta of an option without invoking the BSM model using the binomial tree model. By varying the price of the underlying stock and recalculating the option price, we can numerically approximate the derivative dC/dS using the formula:

$$\frac{dC}{dS} \approx \frac{\text{New Option Price} - \text{Original Option Price}}{\text{New Stock Price} - \text{Original Stock Price}}$$

Create a binomial tree using the following information: $S_0 = 100$, $K = 100$, $n = 10$, $T = 0.1$, $\sigma = 0.3$, and $r = 0.02$, where n is the number of steps, and find the price of the European call option. Vary the stock price and numerically find delta.
We can numerically approximate gamma as well.

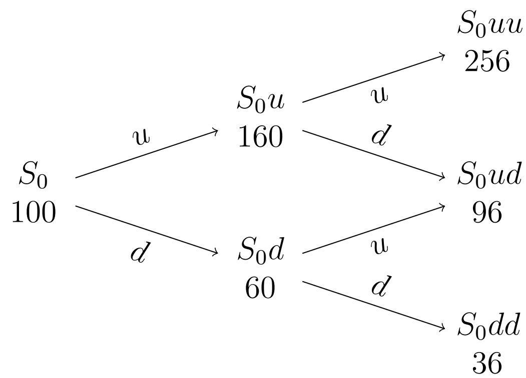
$$\frac{d^2C}{dS^2} \approx \frac{\text{New Delta} - \text{Original Delta}}{\text{New Stock Price} - \text{Original Stock Price}}$$

Find gamma of the option.

Solution: Refer to "MC_Simulation.xlsx".
The price of the European call option is 3.787. In the spreadsheet, we see that the price of the underlying stock in the "up" node at step $i = 1$ is $S_1^u = 103.045$, and the price in the "down" node at $i = 1$ is $S_1^d = 97.045$. The price of the European call option in the "up" node at step $i = 1$ is $c_1^u = 5.381$, and the price in the "down" node at $i = 1$ is $c_1^d = 2.221$. Therefore, the delta is 0.527.
Using the spreadsheet, we can calculate delta for a symmetric 3% increase and decrease in the stock price. For the 3% increase, we plug in $S_0 = 100 \times 1.03 = 103$ and find the delta of the call option to be 0.647. For the 3% decrease, we plug in $S_0 = 100 \times 0.97 = 97$ and find the delta of the call option to be 0.401. The gamma is 0.041.

16 Binomial Tree

Bank XYZ offers an equity option. The expiration date of the option is two periods from now. The option is not initially specified to be a put or a call. Instead, the owner makes this choice after one period. Once the choice is made, the option can be exercised at any time. For example, if after one period the owner chooses for the new exotic option to be a put, it would at that time become identical to an ordinary American put with one period remaining until expiration. A customer has asked for a quote for an option of this type on Stock ABC with a strike price of 100. The current price of Stock ABC is 100 per share, and over each period the stock price evolves as shown on the tree diagram below. The risk-neutral probability of an "up" move is 0.5. The stock does not pay dividends, and the interest rate is 10% per period. What is the lowest price the firm could charge and still break even?



Solution: Let $C(S;n)$ and $P(S;n)$ be the values of American call and put options when the underlying stock price is S and there are n periods remaining until maturity. We begin by finding the payoffs of the call and put options at each node by moving backwards through the tree. Two periods from now, i.e., when $n = 0$, the payoff of a call option will be $256-100 = 156$ in the "up-up" node and 0 in either the "up-down" or "down-up" or "down-down" nodes. Similarly, the payoff of a put option will be 0 in the "up-up" node, $100-96 = 4$ in the "up-down", "down-up" nodes, and $100-36 = 64$ in the "down-down" node.
What would be the values of the call and put options in each node one period from now, i.e., when $n = 1$, given the payoffs in each node at $n = 0$, the risk-neutral probability of an "up" move of 0.5, and an interest rate of 10% per period?
Assume we are in the "up" node at $n = 1$ and the stock price is 160. If we choose to exercise the call option at this node, then its payoff will be $160 - 100 = 60$. Alternatively, if we choose to wait to exercise the call option at $n = 0$, then its value is equal to $(0.5 \cdot 156 + 0.5 \cdot 0)/1.1 = 70.9091$. Thus, the value of the call option in the "up" node at $n = 1$ is $C(160;1) = \max[60, 70.9091] = 70.9091$. A similar calculation for the put option yields a value of $P(160;1) = \max[100-160, (0.5 \cdot 0 + 0.5 \cdot 4)/1.1] = 1.8182$.
Now, assume we are in the "down" node at $n = 1$ and the stock price is 60. If we choose to exercise the call option at this node, then its payoff will be $60 - 100 = -40$. Alternatively, if we choose to wait to exercise the call option at $n = 0$, then its value is equal to $(0.5 \cdot 0 + 0.5 \cdot 0)/1.1 = 0$. Thus, the value of the call option in the "down" node at $n = 1$ is $C(60;1) = \max[-40, 0] = 0$. A similar calculation for the put option yields a value of

$P(60;1) = \max[100-60, (0.5\cdot 4 + 0.5\cdot 64)/1.1] = 40.$
To summarize, the values of the call and put options at $n = 1$ are: $C(160;1) = 70.9091$, $P(160;1) = 1.8182$, $C(60;1) = 0$, $P(60;1) = 40$. Thus, if the stock price goes down to 60 at $n = 1$, the buyer will choose for the option to be a put, and the put will be exercised immediately. If the stock price instead goes up to 160 at $n = 1$, the buyer will choose for the option to be a call, and the call will be held until its expiration.
Given the optimal exercise policy above, we can find the current value of the option, i.e., at $n = 2$, by discounting the expected payoffs from exercising the put option in the "down" node at $n = 1$ and holding the call option in the "up" node at $n = 1$. The current value of the option is: $[0.5\cdot C(160;1) + 0.5\cdot P(60;1)]/1.1 = [0.5\cdot 70.9091 + 0.5\cdot 40]/1.1 = 50.4132.$

17 Option Greeks

All assumptions of the Black-Scholes-Merton option pricing model hold. Stock XYZ is priced at \$30. It has volatility 25% per year. The annualized continuously-compounded risk-free interest rate is 3.0%.

1. Compute the price of a European call option with strike price \$31, which matures in 6 months.
2. Compute the option Delta at time 0.
3. Suppose that at time 0 the stock price changes instantaneously from \$30 to \$34. Compute the resulting change in the option price.
4. Suppose that at time 0 the stock price changes instantaneously from \$30 to \$34. Compute the resulting change in the value of the replicating portfolio for this option.
5. Suppose that at time 0 the stock price changes instantaneously from \$30 to \$26. Compute the resulting change in the option price.
6. Suppose that at time 0 the stock price changes instantaneously from \$30 to \$26. Compute the resulting change in the value of the replicating portfolio for this option.
7. Does the change in the option price exceed the change in the value of its replicating portfolio? If so, why?

Solution:

1. 1.873
2. $N(d_1) = 0.495$
3. $4.409 - 1.873 = 2.536$ (using the BSM)
4. Delta $\cdot(34-30) = 1.98$
5. $0.490 - 1.873 = -1.383$

6. Delta $\cdot(26 - 30) = -1.98$

7. We observe that the change in the option price exceeds the change in the value of its replicating portfolio in both cases. This is due to the positive convexity (Gamma) of the call option.