Practice Problem Set

BUSS386 Futures and Options

1 Option Greeks

A financial institution has just sold 1,000 7-month European call options on the Japanese yen. Suppose that the spot exchange rate is 0.80 cent per yen, the exercise price is 0.81 cent per yen, the risk-free interest rate in the United States is 8% per annum, the risk-free interest rate in Japan is 5% per annum, and the volatility of the yen is 15% per annum. Calculate the delta, gamma, vega, theta, and rho of the financial institution's position. Interpret each number.

2 Portfolio Insurance

A fund manager has a well-diversified portfolio that mirrors the performance of the S&P 500 and is worth $360 million. The value of the S&P 500 is 1,200, and the portfolio manager would like to buy insurance against a reduction of more than 5% in the value of the portfolio over the next 6 months. The risk-free interest rate is 6% per annum. The dividend yield on both the portfolio and the S&P 500 is 3%, and the volatility of the index is 30% per annum.

1. If the fund manager buys traded European put options, how much would the insurance cost?
2. Create alternative strategies involving traded European call options, and show that they lead to the same result.

3 Risk Management

A financial institution has the following portfolio of over-the-counter options on sterling:

Type	Position	Delta of option	Gamma of option	Vega of option
Call	$-1,000$	0.50	2.2	1.8
Call	-500	0.80	0.6	0.2
Put	$-2,000$	-0.40	1.3	0.7
Call	-500	0.70	1.8	1.4

A traded option is available with a delta of 0.6, a gamma of 1.5, and a vega of 0.8.

1. What position in the traded option and in sterling (the underlying asset) would make the portfolio both gamma neutral and delta neutral?

2. What position in the traded option and in sterling would make the portfolio both vega neutral and delta neutral?

4 The Black's Model

Consider a futures. Its current price is \$70.00 and its volatility is 16.70% per annum. Suppose the risk-free interest rate is 5.00% per annum (continuous compounding). Use the Black's model to calculate the value of a five-month European put on the futures with a strike price of \$65.00.

5 Delta

What does it mean to assert that the delta of a call option is 0.7? How can a short position in 1,000 options be made delta neutral when the delta of each option is 0.7?

6 Delta

Calculate the delta of an at-the-money six-month European call option on a non-dividend-paying stock when the risk-free interest rate is 10% per annum and the stock price volatility is 25% per annum. Use the BSM model.

7 Portfolio Insurance

Why did portfolio insurance not work well on October 19, 1987?

8 Delta of Futures Option

What is the delta of a short position in 1,000 European call options on silver futures? The options mature in eight months, and the futures contract underlying the option matures in nine months. The current nine-month futures price is \$8 per ounce, the exercise price of the options is \$8, the risk-free interest rate is 12% per annum, and the volatility of silver futures prices is 18% per annum. Hint: Use Black's model.

Extra: If you want to hedge the option position using the underlying asset (i.e., silver), how many units of silver do you need? (Assume no storage cost for silver)

9 Delta and Gamma

A company uses delta hedging to hedge a portfolio of long positions in put and call options on a currency. Which of the following would give the most favorable result?

1. A virtually constant spot rate
2. Wild movements in the spot rate

Explain your answer.

10 Gamma and Vega

Under what circumstances is it possible to make a European option on a stock index both gamma neutral and vega neutral by adding a position in one other European option with the same underlying asset but different K and T? Use the BSM model.

11 The Put-Call Parity and Greeks

Use the put-call parity relationship to derive, for a non-dividend-paying stock, the relationship between:

1. The delta of a European call and the delta of a European put.
2. The gamma of a European call and the gamma of a European put.
3. The vega of a European call and the vega of a European put.
4. The theta of a European call and the theta of a European put.

12 Hedging using Greeks

A financial institution has the following portfolio of over-the-counter options on sterling:

Type	Position	Delta of Option	Gamma of Option	Vega of Option
Call	$-1,000$	0.5	2.2	1.8
Call	-500	0.8	0.6	0.2
Put	$-2,000$	-0.40	1.3	0.7
Call	-500	0.70	1.8	1.4

A traded option is available with a delta of 0.6, a gamma of 1.5, and a vega of 0.8.

1. What position in the traded option and in sterling would make the portfolio both gamma neutral and delta neutral?
2. What position in the traded option and in sterling would make the portfolio both vega neutral and delta neutral?

13 Portfolio Insurance

Suppose that \$70 billion of equity assets are the subject of portfolio insurance schemes. Assume that the schemes are designed to provide insurance against the value of the assets declining by more than 5% within one year. Suppose $S_0 = 70$, $K = 66.5$, $T = 1$. Other parameters are estimated as $r = 0.06$, $\sigma = 0.25$, and $q = 0.03$. Calculate the value of the stock or futures contracts that the administrators of the portfolio insurance schemes will attempt to sell if the market falls by 23% in a single day.

14 Hedging with Delta and Gamma

A bank's position in options on the dollar-euro exchange rate has a delta of 30,000 and a gamma of $-80,000$. Explain how these numbers can be interpreted. The exchange rate (dollars per euro) is 0.90. What position would you take to make the position delta neutral? After a short period of time, the exchange rate moves to 0.93. Estimate the new delta. What additional trade is necessary to keep the position delta neutral? Assuming the bank did set up a delta-neutral position originally, has it gained or lost money from the exchange-rate movement?

15 Option Greeks and Binomial Tree

According to the Black-Scholes-Merton (BSM) model, the delta of a European option on a non-dividend-paying stock has the analytical expressions. We can also compute the delta of an option without invoking the BSM model using the binomial tree model. By varying the price of the underlying stock and recalculating the option price, we can numerically approximate the derivative dC/dS using the formula:

$$\frac{dC}{dS} \approx \frac{\text{New Option Price} - \text{Original Option Price}}{\text{New Stock Price} - \text{Original Stock Price}}$$

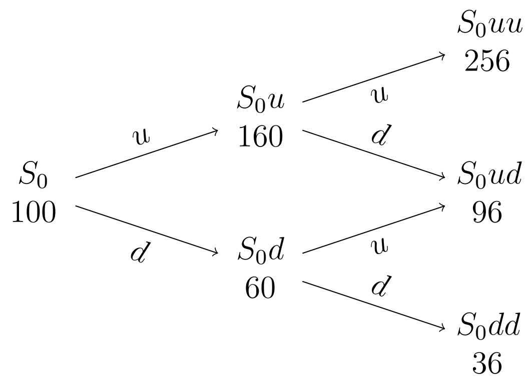
Create a binomial tree using the following information: $S_0 = 100$, $K = 100$, $n = 10$, $T = 0.1$, $\sigma = 0.3$, and $r = 0.02$, where n is the number of steps, and find the price of the European call option. Vary the stock price and numerically find delta.
We can numerically approximate gamma as well.

$$\frac{d^2C}{dS^2} \approx \frac{\text{New Delta} - \text{Original Delta}}{\text{New Stock Price} - \text{Original Stock Price}}$$

Find gamma of the option.

16 Binomial Tree

Bank XYZ offers an equity option. The expiration date of the option is two periods from now. The option is not initially specified to be a put or a call. Instead, the owner makes this choice after one period. Once the choice is made, the option can be exercised at any time. For example, if after one period the owner chooses for the new exotic option to be a put, it would at that time become identical to an ordinary American put with one period remaining until expiration. A customer has asked for a quote for an option of this type on Stock ABC with a strike price of 100. The current price of Stock ABC is 100 per share, and over each period the stock price evolves as shown on the tree diagram below. The risk-neutral probability of an "up" move is 0.5. The stock does not pay dividends, and the interest rate is 10% per period. What is the lowest price the firm could charge and still break even?



17 Option Greeks

All assumptions of the Black-Scholes-Merton option pricing model hold. Stock XYZ is priced at \$30. It has volatility 25% per year. The annualized continuously-compounded risk-free interest rate is 3.0%.

1. Compute the price of a European call option with strike price \$31, which matures in 6 months.

2. Compute the option Delta at time 0.

3. Suppose that at time 0 the stock price changes instantaneously from \$30 to \$34. Compute the resulting change in the option price.

4. Suppose that at time 0 the stock price changes instantaneously from \$30 to \$34. Compute the resulting change in the value of the replicating portfolio for this option.

5. Suppose that at time 0 the stock price changes instantaneously from \$30 to \$26. Compute the resulting change in the option price.

6. Suppose that at time 0 the stock price changes instantaneously from \$30 to \$26. Compute the resulting change in the value of the replicating portfolio for this option.

7. Does the change in the option price exceed the change in the value of its replicating portfolio? If so, why?