Practice Problem Set: Solutions

BUSS386 Futures and Options

1 Option Payoffs

The payoff to the investor is:

$$\max(0, K - S_T) - \max(0, S_T - K) = K - S_T$$

This is the same as a short position in a forward contract with delivery price K.

2 Margin Account

When an investor buys an option, cash must be paid upfront. There is no possibility of future liabilities. When selling options, potential future liabilities arise, and margin is required to protect against default.

3 Stock Splits

The strike price is reduced to $30, and the option allows purchase of twice as many shares.

4 Employee Stock Options

Exercise of employee stock options leads to new shares being issued, changing equity structure. Exchange-traded options do not result in new shares.

5 Option Mechanics

$$\text{Forward payoff: } S_T - F_0$$

$$\text{Put payoff: } \max(F_0 - S_T, 0)$$

$$\text{Total payoff: } \max(F_0 - S_T, 0) + S_T - F_0 = \max(0, S_T - F_0)$$

which is the value of a European call with strike F_0.

6 Options vs. Forwards

Forwards lock in exchange rates, eliminating uncertainty, but can result in unfavorable outcomes. Options are costlier (premium) but provide downside protection while allowing upside.

7 Option Mechanics (Adjustments)

(a) Adjusted to $500 \times 1.1 = 550$ shares at \$36.36.

(b) No adjustment. Option contracts are not adjusted for cash dividends.

(c) Adjusted to $500 \times 4 = 2000$ shares at \$10.

8 American Call Options

Delaying exercise delays the payment of the strike price, allowing the holder to earn interest on K for longer. Even if interest rates are zero, delaying exercise preserves the option's insurance value. The holder benefits from upside while avoiding downside risk. If the option is exercised early, its value at expiration is S_T; delaying exercise yields $\max(K, S_T)$.

9 Put–Call Parity

$$p = c + Ke^{-rT} - S_0 = 1 + 20e^{-0.04\times0.25} - 19 = 1.80$$

10 Option Price Bounds and Arbitrage

Lower bound violated. Buy call, short stock. Arbitrageur receives \$59 now $(64 - 5)$, invests \$0.79 for one month to cover dividend, and invests remaining \$58.21 for four months. Present value of gain: \$5.56 - \$5.00 = \$0.56 in all cases.

11 American Put Options

Early exercise is attractive when interest earned on K exceeds insurance value lost. Higher rates increase this benefit; lower volatility reduces insurance value, making early exercise more appealing.

12 Options and Capital Structure

(a) Manager's value: $\max(V - D, 0)$ — a call on V with strike D.

(b) Debt holders: $\min(V, D) = D - \max(D - V, 0)$. Equivalent to a risk-free bond plus a short put on V.

(c) Manager can increase position value by increasing both V and σ_V. Higher volatility benefits option-like payoffs.

13 Bear Spread

Using calls: short low-strike, long high-strike (same maturity). Using puts: short low-strike, long high-strike. Both yield limited downside exposure.

14 Butterfly Spread

Buy \$15 and \$20 calls, sell two \$17.5 calls. Initial cost: \$4 + \$0.5 - 2(\$2) = \$0.5.

Stock Price S_T	Profit
$S_T < 15$	-0.5
$15 < S_T < 17.5$	$S_T - 15 - 0.5$
$17.5 < S_T < 20$	$20 - S_T - 0.5$
$S_T > 20$	-0.5

15 Strangle

Stock Price S_T	Profit
$S_T < 45$	$45 - S_T - 5$
$45 < S_T < 50$	-5
$S_T > 50$	$S_T - 50 - 5$

16 Bull Spread and Put–Call Parity

Let p_1, p_2 and c_1, c_2 denote puts and calls with strikes $K_1 < K_2$. From put–call parity:

$$p_1 - p_2 = c_1 - c_2 - (K_2 - K_1)e^{-rT}$$

Hence, the initial investment from puts is lower by $(K_2 - K_1)e^{-rT}$ than that from calls. The call-based strategy's profit exceeds the put-based one by $(K_2 - K_1)(1 - e^{-rT})$.

17 Butterfly Spread and Put–Call Parity

Let c_1, c_2, c_3 and p_1, p_2, p_3 denote call and put prices with strikes K_1, K_2, K_3. By parity:

$$c_i + K_i e^{-rT} = p_i + S_0, \quad i = 1, 2, 3$$

Hence,

$$c_1 + c_3 - 2c_2 + (K_1 + K_3 - 2K_2)e^{-rT} = p_1 + p_3 - 2p_2$$

Since $K_2 - K_1 = K_3 - K_2$, we have $K_1 + K_3 - 2K_2 = 0$, so

$$c_1 + c_3 - 2c_2 = p_1 + p_3 - 2p_2$$

Thus, both spreads cost the same.

18 Straddle Payoffs

A straddle is created by buying both options. Total cost = \$10.

Stock Price S_T	Profit
$S_T < 60$	$S_T - 70$
$S_T > 60$	$50 - S_T$

The strategy results in a loss when $50 < S_T < 70$.

19 Forward from Options

Buy a European call and sell a European put with same strike K and maturity T. The payoff is $S_T - K$, identical to a long forward. If $K = F_0$, both options are equally priced and the synthetic forward has zero value.

20 Portfolio Payoffs

Profit diagrams are shown below. In each, the dotted line shows component payoffs, and the solid line shows total profit.