Introduction to Swaps

BUSS386. Futures and Options

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Lecture Outline

- Interest Rate Futures
  - Treasury bond futures
  - Eurdollar and SOFR futures
- Swaps
  - Products, pricing and risk management applications
  - Currency, commodity, total rate of return swaps
- Reading: §6.1–6.3 and Ch. 7

Day Counting

Day Count Conventions

- Pricing in financial markets started long before computers...
  - People in different countries took different strategies to ease the calculation of accrued interests over time
  - 30 days per month? 360 or 365 days per year?
- Conventions vary from country to country and from instrument to instrument
  - Actual/Acutal: US treasury bonds, Australia
  - 30/360 method: US corporate/municipal bonds, Eurobonds
  - Actual/360: US money market
  - Actual/365: Korea, UK, Japan

  - X/Y, where X is the number of days in a month, and Y is the number of days in a year.

    [Source: https://www.rbcits.com/en/gmi/global-custody/market-profiles.page]

Day Count Conventions: Example

- Consider a Treasury bond and a corporate bond both have the same annual coupon payment dates (Principal: \$100, coupon rate: 8%).
  - Their last coupon payment date is March 1, 2018, and the next coupon date is September 1, 2018.
- How much interest is accured for the period from March 1, 2018 to July 3, 2018, for the two bonds, respectively?
  - Act/Act: $\frac{124}{184} \times \$4 = 2.6957$
  - 30/360: $\frac{122}{180} \times \$4 = 2.7111$
- How about from October 3, 2018 to January 1, 2019?
  - Act/Act: $\frac{92}{181} \times \$4 = 2.0331$
  - 30/360: $\frac{90}{180} \times \$4 = 2.0000$
- Excel functions: `Days` and `Days360`

Day Count Conventions: Example

- What if we use Actual/365?
  - Divide 8% by 365 = 0.02191%
  - Multiply by # of days from March 1 to July 3, 2018 (124) = 2.7178%
- Actual/360?
  - Divide 8% by 360 = 0.02222%
  - Multiply by # of days from March 1 to July 3, 2018 (124) = 2.7555%

NB Therefore, 8% in Actual/360 is equivalent to $8\% \times \frac{365}{360} = 8.1111\%$

- Divide 8.1111% by 365 = 0.02222%
- Multiply by # of days from March 1 to July 3, 2018 (124) = 2.7555%

NB 1% in Actual/360 would earn $1\% \times 365/360$ of interest in 365 days.

Interest Rate Futures

Treasury Futures

- Underlying variable: "Virtual" bond price or interest rate
  - Treasury bonds, 10-year, 5-year, 2-year Treasury notes, with 6% coupon rate in U.S., 5% in Korea (since 1999) $\Rightarrow$ Prices
  - Federal funds rates/Eurodollar/SOFR futures $\Rightarrow$ Interest rates

- In the U.S., the bonds actually delivered by the party on the short side of the futures contract are not necessarily of those exact same maturities.
  - Acceptable government bonds/notes to deliver are:
    - Ultra T-Bond: 25 years $<$ maturity
    - Treasury Bond: $15 \leq$ maturity ≤ 25 years
    - 10-year Treasury note: $6.5 <$ maturity ≤ 10
    - 5-year Treasury note ≈ 5
    - 2-year Treasury note ≈ 2

- In Korea, TB futures are cash-settled.[1]

[1] KRX designates a basket of 3 eligible delivery bonds.

Treasury Futures: Quotes

Table 6.1 Futures quotes for a selection of CME Group contracts on interest rates on May 21, 2020.

	Open	*High*	*Low*	*Prior settlement*	*Last trade*	*Change*	*Volume*
Ultra T-Bond, $100,000							
June 2020	220-06	221-31	220-06	220-17	220-28	+0-11	238,736
Sept. 2020	218-25	220-12	218-25	218-31	219-14	+0-15	137,715
Treasury Bond, $100,000							
June 2020	179-15	180-08	179-13	179-20	179-27	+0-07	395,908
Sept. 2020	177-29	178-22	177-29	178-03	178-08	+0-05	211,246

- In the U.S., thirty-seconds of a dollar per $100 face value. In Korea, decimals.
  - $220'06 = 220 + 6/32 = 220.1875$
  - $134'215 = 134 + 21.5/32 = 134.671875$
  - $124'1525 = 124 + 15.25/32 = 124.4765625$

Treasury Bond Quotes

- The quoted price is for a bond with a face value of \$100.
- Example: a quote of 90'05 indicates that if the bond has a face value of \$100,000 its price will be $(90 + 5/32) \times 1,000 = \$90,156.25$
- This quoted price is also known as the clean price.
- The actual cash price that has to be paid by the purchaser of the bond is known as the dirty price.

  Cash price = Quoted price + Accrued Interest

NB In practice, one first needs to compute cash price and adjust for accrued interest to get quoted price.

Treasury Bond Quotes: Example

- On March 5th 2013, there is a 11%-coupon treasury bond maturing on July 10th 2028, with a quoted price of 95'16.

- Coupons semi-annually: last coupon was paid on January 10th 2013. The next coupon date is July 10th 2013.
  - The actual number of days between Jan 10 and Mar 5 is 54.
  - The actual number of days between Jan 10 and Jul 10 is 181.
  - Each coupon pays $\$100 \times 0.11/2 = \$5.50$ (on Jan 10 and Jul 10)

- The accrued interest on Mar 5 is: $\$5.50 \times 54/181 = \$1.64$

- The cash price per \$100 face value is thus $\$95.50 + \$1.64 = \$97.14$

- The cash price of a \$100,000 face value bond is thus: \$97,140

Treasury Futures: Settlement

- The Treasury bond futures contract allows the party with the short position to choose to deliver any government bond with a maturity left between 15 and 25 years.

  Cash price = Quoted Price + Accrued interest
  (Quoted price is called settlement price for futures.)
  Cash price = Settlement Price + Accrued interest
  (But the delivered bond may not be a 6% coupon bond.)
  Cash price = Settlement Price × Conversion factor + Accrued interest

- Example
  - Settlement price: 120'00
  - Conversion factor: 1.3800
  - Accrued interest: \$3 per \$100 face value.
  - The cash received by the party with the short position (per \$100 face value)

$$(120.99 \times 1.3800) + 3.00 = \$168.60$$

  - The actual price $= \$168.60 \times 1,000 = \$168,600$ where face value is \$100,000.

Cheapest-to-Deliver Bond

- At any given time during the delivery month, there are many bonds that can be delivered in bond futures contracts
- The party with the short position, when delivering the bond, receives:
  ① Settlement Price $\times$ Conversion factor $+$ Accrued interest
- The cost of purchasing the bond is:
  ② Quoted bond price $+$ Accrued interest
- Thus the cheapest-to-deliver bond is the one minimizing: ② - ①
  Quoted bond price $-$ (Settlement price $\times$ Conversion factor)

Cheapest-to-Deliver Bond: Example

- The party with the short position has decided to deliver and is trying to choose between the three bonds in the table below. Assume the most recent settlement price is 93-08, or 93.25.

Bond	Quoted price	Conversion factor
1	99.50	1.0382
2	143.50	1.5188
3	119.75	1.2615

- The cost of delivering each of the bonds is as follows:

  Bond 1: $99.50 - 93.25 \times 1.0382 = \$2.69$
  Bond 2: $143.50 - 93.25 \times 1.5188 = \$1.87$
  Bond 3: $119.75 - 93.25 \times 1.2615 = \$2.12$

  The cheapest-to-deliver bond is Bond 2.

Treasury Bond Futures: Conversion factor

- The conversion factor is the price of the bond if it were priced to yield 6%.
- Example: Bond A is a 7% coupon bond with exactly 8 years to maturity, a price of 103.71, and a yield of 6.4%. This bond would have a price of 106.28 if its yield were 6%. Thus its conversion factor is 1.0628.
- Example: Bond B has 7 years to maturity and a 5%coupon. Its current price and yield are 92.73 and 6.3%. It would have a conversion factor of 0.9435, since that is its price at a 6% yield.

Treasury Bond Futures: Conversion factor - Intuition

- The Treasury bond futures contract is built around a *benchmark* 6% annual coupon. The conversion factor (CF) tells us how to scale the futures price so that, under a 6% world, different deliverable bonds are treated fairly.

- If the bond's coupon is exactly 6%:
  - Its cash flows, discounted at 6%, line up perfectly with the benchmark.
  - $\Rightarrow$ CF $= 1$. The short delivers the bond and gets the futures invoice price (up to accrued interest).

- If the bond's coupon is higher than 6%:
  - It pays *more* than the benchmark each year.
  - $\Rightarrow$ Its value at a 6% discount rate is *higher*, so CF > 1. The invoice price is scaled up to compensate the short for delivering a richer coupon stream.

- If the bond's coupon is lower than 6%:
  - It pays *less* than the benchmark each year.
  - $\Rightarrow$ Its value at a 6% discount rate is *lower*, so CF < 1. The invoice price is scaled down because the short delivers a leaner coupon stream.

- **Bottom line:** Coupons $>$6% get CFs $>$1; coupons $<$6% get CFs $<$1; coupons $=$6% get CF $=$1.

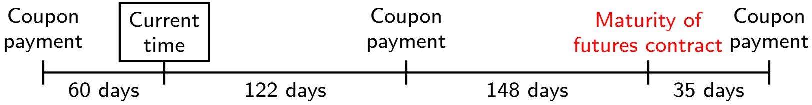
Treasury Bond Futures: Why designed this way?

- **Why not name one specific T-bond?** If a single bond were the underlying, it could become scarce (or be "cornered"). Shorts might not be able to find that bond to deliver.
- **Then why not cash-settle to a bond index (like the S&P 500)?** The futures would track the *index*, not any particular bond you hold. That creates **basis risk**: your hedge can move differently from your actual bond.
- **What the contract actually does:** Allow delivery of a **basket** of eligible Treasury bonds and use **conversion factors** to level the playing field. The short chooses which bond to deliver. This keeps the futures tied to real deliverable bonds and reduces squeeze risk.
- **Bottom line:** The design balances *good hedging* (deliverable bonds) with *market robustness* (no single-bond squeeze, lower basis risk than an index-only design).

Determining the Futures Price

- Exact theoretical futures prices are difficult to determine because there are many factors that affect the futures price:
  - Delivery can be made any time during the delivery month
  - Any of a range of eligible bonds can be delivered
- Assume, for simplicity, that the cheapest-to-deliver bond and the delivery date are known. Then $F_0 = (S_0 - I)e^{rT}$
  - I is the present value of the coupons during the life of the futures contract,
  - T is the time until the futures contract matures, and
  - r is the risk-free interest rate applicable to a time period of length T.

Determining the Futures Price: Example



- Suppose we know the cheapest-to-deliver bond will be a 12% coupon bond with a conversion factor of 1.6000, and delivery will be in 270 days.
- The last coupon date was 60 days ago, the next coupon date is in 122 days, and the coupon date thereafter is in 305 days.
- Interest rate is 10% per annum (continuous compounding). The quoted bond price is $115.
- The cash price of the bond is $115 + \frac{60}{60+122} \times 6 = 116.978$
- The present value of a coupon in 122 days is $6e^{(-0.1)(122/365)} = 5.803$.
- The futures value is $(116.978 - 5.803)e^{(0.1)(270/365)} = 119.711$.
- At delivery, the settlement futures price (12%) is calculated by subtracting the accrued interest $Cash = Q \times CF + AI$
  $119.71 = Q \times 1.6 + \frac{148}{35+148} \times 6 \Rightarrow Q = \frac{114.859}{1.6000} = 71.79$

Eurodollar Futures

- The 3-month Eurodollar futures contract is the most popular interest rate futures contract.
  - A Eurodollar is a dollar deposited in a bank outside the U.S.
  - A 3-month Eurodollar futures contract is a futures contract on the interest that **will be paid** (by someone who borrows at the Eurodollar interest or deposit rate, same as 3-month LIBOR rate) on \$1 million for a future period of 3 months.
  - A change of one basis point or 0.01 in a Eurodollar futures quote corresponds to a contract price change of $\$25 = \$1,000,000 \times 0.0001 \times (3/12)$
- A Eurodollar futures contract is settled in cash
  - When it expires the final settlement price is: $100 - R$, where R is the actual 3-month Eurodollar interest rate on that day.
  - R is expressed with quarterly compounding.
  - R uses an (actual/360) day count convention.

Eurodollar Futures: Example

- An investor wants to lock in the interest rate for a 3-month period starting Sep 16th 2015, for \$100 million (to invest).
- Today, the Sep 2015 Euro futures quote is 96.500, meaning that the investor can lock in a rate of $100 - 96.5 = 3.5\%$ per annum.
- The investor hedges by buying 100 contracts.
- On Sep 16th 2015, the 3-month Eurodollar quote is 97.400.
  - The difference in basis points is $100 \times (97.40 - 96.50) = 90$
  - The investor gains: $100 \times 25 \times 90 = 225,000$ on the Eurodollar futures.
  - The interest earned on the 3-month investment is: $100,000,000 \times (100 - 97.4)/100 \times (3/12) = 650,000$
  - Once we add the futures gains to the interest, the total earned is: $650,000 + 225,000 = 875,000$
  - Checking that 3.5% was indeed locked in: $100,000,000 \times (100 - 96.5)/100 \times (3/12) = 875,000$

SOFR Futures

- Three-month SOFR futures and Eurodollar futures contracts are very similar.
- Main difference: The Eurodollar futures contract is settled **at the beginning** of the three-month period to which the rate applies whereas the three-moth SOFR futures contract is settled **at the end** of the three-month period.
  - SOFR futures settlement equals 100 minus the result of compounding one-day SOFR rates over the previous three months
  - Suppose that the (annualized) SOFR overnight rate on the ith business day of a period is r_i and the rate applies to d_i days. The (annualized) interest rate for the period is

$$[(1+r_1\hat{d}_1)(1+r_2\hat{d}_2)\dots(1+r_n\hat{d}_n)-1]\times\frac{360}{D},$$

where $\hat{d}_i = d_i/360$ and $D=\sum d_i$ is the number of days in the period. On most days $d_i = 1$[2]

[2] But weekends and holidays lead to the overnight rates being applied to more than one day. For example, on a Friday d_i will normally be equal to 3.

SOFR Futures: Example

- Suppose that on May 21, 2020, an investor has agreed to pay the three-month SOFR rate plus 200 basis points on \$100 million of borrowings for three months beginning on December 16, 2021.
- The December 2021 three-month SOFR futures is 99.990 (i.e., SOFR rate$=$0.01%).
- She uses futures to lock in a borrowing rate of $0.01\% + 2\%$.
  - Short 100 December contracts (\$1m each).
  - Suppose that the final settlement proves to be 99.200.
  - The futures contract has declined by 79 basis points (from 99.990 to 99.200).
  - The gain is $100 \times \$25 \times 79 = \$197,500$ on 100 contracts.
  - This amount is settled in March 2022.

Swaps

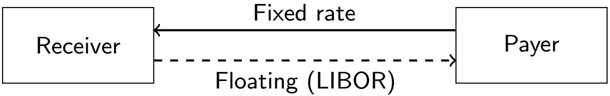
Swap basics

- A swap is a contract calling for an exchange of payments, on one or more future dates, determined by the difference in two reference prices or interest rates.
- A single-payment swap is equivalent to a cash-settled forward contract.
- A swap provides a means to hedge or speculate on a stream of risky cash flows.
- Traded in over-the-counter market.

Structure of a "Plain Vanilla" Interest Rate Swap

- In the most common type of interest rate swap (fixed for floating), fixed interest rate payments are exchanged for floating interest rate payments at regular intervals over the life of the contract.

- No principal is exchanged.



LIBOR is the London Interbank Offer Rate. For many years it was the most common reference floating rate for swaps. It is being replaced by other reference rates, e.g., SOFR.

- An interest rate swap can also be described as a package of forward rate agreements (FRAs)
  - A forward rate agreement is a one-time exchange based on a fixed interest rate and a floating one

Swaps - Terminology

- Notional Principal: Amount of principal upon which the interest payments are based. This principal may not be exchanged.

- Counterparties: The two participants in the swap.

- Payer, receiver?!? Be careful...
    - Fixed Rate Payor: The counterparty who pays a fixed rate, and receives a floating rate in the swap. The fixed rate payor is said to have "bought the swap" or is long in the swap.
    - Floating Rate Payor: The counterparty who pays a floating rate, and receives a fixed rate in the swap. The floating rate payor is said to have "sold the swap" or is short in the swap.

Organization of Trading

- Financial institutions act as market makers and provide bid and ask quotes for the fixed rates that they are prepared to exchange in swaps.

Table 7.4 Example of bid and ask fixed rates in the swap market for a swap where payments are exchanged quarterly (percent per annum).

Maturity (years)	*Bid*	*Ask*	*Swap rate*
2	2.97	3.00	2.985
3	3.05	3.08	3.065
4	3.15	3.19	3.170
5	3.26	3.30	3.280
7	3.40	3.44	3.420
10	3.48	3.52	3.500

- Bid: fixed rate that the market maker pays to receive floating
- Ask: fixed rate that the market maker receive to pay floating.
- Swap rate: the average of the bid and ask rates
- The spread compensates the market maker for its costs.

Interest rate swap pricing

- For floating rate payor, swap is initially equivalent to going long in a fixed rate bond priced at par, and going short in a floating rate bond priced at par
- For the fixed rate payor, the equivalent cash position is the opposite
- In general, we price a swap by finding the difference between the present value of the fixed and floating rate payments.

Swap Pricing: Valuing a Floating Rate Bond

- Floating rate bonds always are priced at par at reset dates.

Proof Let r_T be the one-period reset rate realized at time T.

- We find the price at time 0 by working backwards.
- At time $T-1$, there is one remaining payment of principal and interest, equal to $Fe^{r_{T-1}}$. Its value at time $T-1$, $P_{T-1} = Fe^{r_{T-1}}e^{-r_{T-1}} = F$.
- Stepping back to time $T-2$, $P_{T-2} = Fe^{r_{T-2}}e^{-r_{T-2}} = F$.
- Continuing in this way, it is clear that the price equals the face value on all reset dates, including at time 0.
- Assume that reset frequency is annual.

Swap Pricing: A No-Arbitrage Condition

- At swap initiation, the present value of the fixed and floating rate payments must be equal.
  - That is because entering into a swap is free, and a voluntary exchange has to be fair to both sides.

- Because we know that the present value of the floating payments equals the face value of the floating rate bond, the present value of the fixed rate payments also must equal the face value of the fixed rate bond.

- Thus, the fixed rate on the swap is determined by setting the present value of the future fixed rate payments equal to par.

Swap Pricing: Implementation

- Imagine that you have derived a spot yield curve $Y_1, Y_2, \ldots, Y_T$ that is appropriate for discounting the fixed rate swap payments

- Then the coupon rate on the swap solves:

$$F = cFe^{-Y_1} + cFe^{-2Y_2} + ... + F(1+c)e^{-TY_T}$$

- Assume that reset frequency is annual.

$$c = \frac{1 - e^{-TY_T}}{e^{-Y_1} + e^{-2Y_2} + ... + e^{-TY_T}}$$

- Note:
  - Swaps are priced to be consistent with the yield curve and hence with implied forward rates, FRAs, and other interest rate forwards and futures
  - Over time the value of the swap changes with market interest rates. Like forward contracts, it is zero sum across the two counterparties.

Example: Hedging bank balance sheet risk

- It is December 2025. Southwest savings bank is expanding its holdings:
  - It funds \$1mm of new 10-year mortgages using 3-month time deposits.
  - The current mortgage rate is 10% per year, fixed.
  - The current 3-month rate on time deposits is 8%.
- A profit of 2% is locked in over the first three months.
- After that, the bank bears the risk that interest rates might rise. This risk can be hedged with futures contracts, or more effectively, with an interest rate swap

Example (cont'd)

- Imagine that Southwest can enter into an interest rate swap with the following terms:
  - Maturity = 10 years
  - Fixed rate payor = Southwest S&L.
  - Fixed Rate = 8.65%.
  - Floating Rate = LIBOR
  - Payment Frequency = Semiannual for both fixed and floating.
- Now Southwest can use the fixed (10%) payments from the mortgages to meet their obligations in the swap.
- The floating rate payments received in the swap will be used to pay interest on the deposits backing the mortgages.

Example (cont'd)

- The advantages over the strip of futures contracts include:
  - only one contract
  - covers the entire 10 years
  - avoids illiquid contracts for long-dated futures
  - the timing is more flexible
- But this swap is not a perfect hedge for Southwest:
  - Mortgages are usually amortized over their lifetime, so that the principal balance is declining
  - The frequency of the mortgage payments (often monthly) does not match the semiannual frequency of the fixed payments on a plain vanilla swap
  - The three month rate paid to depositors does not match the six month LIBOR rate received in the swap
  - Mortgages can usually be prepaid

Customized Swap Contracts

- Those features of mortgages, which make a plain vanilla swap a less-than perfect hedge for Southwest, is an example of why there is a demand for more specialized swap products such as:
  - Amortizing Swaps
  - Basis Swaps: exchange two floating rates
  - Swaptions: option to enter a swap
- Specialized swaps tend to be more expensive than a plain vanilla swap and a counterparty may be harder to locate

Why does IRS exist?

The comparative advantage argument

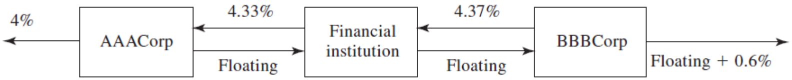
- Consider a 5-year IRS:

	Fixed rate	*Floating rate*
AAACorp	4.0%	Floating −0.1%
BBBCorp	5.2%	Floating +0.6%

  - In fixed rate: AAA saves 0.8% more
  - In floating rate: AAA saves 0.7% more
  - AAA is "comparatively" advantageous in fixed rate.

  1. When AAA uses fixed and BBB uses floating, total borrowing cost is 4.0% + Floating + 0.6%
  2. It is 5.2% + Floating − 0.1% when AAA uses floating and BBB uses fixed.

  - Choosing 1, we save 0.5%. Create an IRS and split the surplus.



Why does IRS exist? (Cont'd)

- The IRS market has been in existence for a long time, we might reasonably expect these types of differences to have been arbitraged away. Then why continue to exist?

- The fixed rate is valid for 5 years.

- However, the spread over the floating reference rate (LIBOR or overnight rate) is adjusted periodically (say, 3-month) after the lender's review.
    - If the credit quality deteriorates, the lender would charge a higher spread.

- The credit spread is increasing in maturity.
    - AAA−BBB spread is wider in fixed rate (5-year) than in floating (3-month)

- BBB's borrowing cost is likely higher than $4.97\% (= 4.37\% + 0.6\%)$, which is valid until the next reset date.
    - The expected cost of borrowing for the next 5-year should be 5.2%.
    - In other words, the market expects that the spread over floating rate will rise for BBB.

Currency Swap

- A currency swap is an agreement to periodically exchange a payment in one currency for a payment in a second currency
  - Payments can be fixed for fixed, fixed for floating, or floating for floating
  - Fixed for fixed is like a portfolio of forward currency contracts
- Example
  - A US exporter is due to receive €5m in 5 equal installments, every 6 months for 2.5 years.
  - The US company can enter into five forward (or futures) contracts to hedge each installment as a stand-alone cash flow.
  - Suppose $S_0 = 1.2673$, $r_\$ = 5\%$, $r_e = 3\%$ (flat term structure in both countries)
  - Using $F = S_0 e^{(r_\$ - r_e)T}$ implies the forward rate schedule:

Maturity	0.5	1	1.5	2	2.5
Forward rate	1.2800	1.2929	1.3059	1.3190	1.3323

Currency swap example (cont'd)

- Alternatively, the US firm can enter into a currency swap
- For instance, the swap contract between the US firm and a bank may be specified, **hypothetically**, as:
  - US firm pays bank €1 mil on $t = 0.5, 1, \ldots, 2.5$
  - Bank pays US firm €1 mil $\times$ \$K/€(where K is the swap rate, say $K = 1.306$) on the same dates.
- What is the net \$ cash flow for the U.S. firm from the swap at any payment date?
  - At every t, the firm receives \$1 mil $\times$ K, and must pay \$1 mil $\times$ S_t dollars/euro (cash settled)
  - Net amount received in the swap is \$1 mil $\times$ $(K - S_t)$. Only this amount is settled.
- U.S. firm also sells Euros received for dollars at current spot rate S_t. Net \$ cash flow $= 1$ mil $\times$ K
  - Note that the notional value, €1 mil, is not delivered to the bank.

Currency swap example (cont'd)

- How is the swap rate K determined?
- The swap rate K is chosen at time 0 so that the value of the swap is equal to zero, i.e., no exchange of money at inception but only in the future.
  - It is determined by a no-arbitrage condition between the forward and swap markets
- The firm receives \$K mil and pays €1 mil at t, i.e. $\$K - €1 \times F_t$.
  - PV of cash flows $\$ \times (e^{-r_\$ 0.5}(K - F_{0.5}) + ... + e^{-r_\$ 2.5}(K - F_{2.5}))$
- Set K so that present value of portfolio equals zero, and hence swap value is zero.

$$K = w_{0.5} F_{0.5} + ... + w_{2.5} F_{2.5}$$

$$w_t = \frac{e^{-r_\$ t}}{e^{-r_\$ 0.5} + ... + e^{-r_\$ 2.5}}$$

Currency swap example (cont'd)

- We obtain an alternative (equivalent) formulation by substituting the forward prices

$$F = S_0 e^{(r_\$ - r_e)T}$$

- Then we have a currency swap rate

$$K = S_0 \frac{e^{-r_e 0.5} + ... + e^{-r_e 2.5}}{e^{-r_\$ 0.5} + ... + e^{-r_\$ 2.5}}$$

  - PV of \$K for 2.5 years = PV of €1 for 2.5 years in \$ term
  - The currency swap rate equals the current exchange rate multiplied by the ratio of the relative risk-free borrowing costs in the two currencies
    - If \$ borrowing is more costly (i.e., the ratio > 1), then $K > S_0$

Hedging with swaps versus forwards

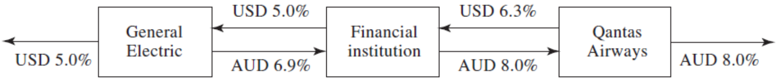
- The payoff profile from the sequence of forwards and one swap is different:
    - The sequence of forwards implies the US firm gets less money early on, and more later on (from \$1.28 mil to \$1.3323 mil)
    - The swap implies the firm gets a constant amount \$1.306 mil every payment
- Both strategies perfectly hedge the exposure, as the exchange rate risk is eliminated and both payoff profiles are known at 0. And both have the same present value.
- Differs in liquidity, transaction costs, etc.

Why does currency swap exist?

- The comparative advantage argument makes sense here.

	USD	*AUD*
General Electric	5.0%	7.6%
Qantas Airways	7.0%	8.0%

- Possible sources of comparative advatange
  - Taxes
  - Reputation



Commodity swaps

- A commodity swap is an agreement to periodically exchange a pre-specified fixed payment for a payment linked to the market price of a commodity
  - Usually contract calls for cash-settlement but in principle could require physical deliveries

- Example of commodity swap
  - A company needs to buy 100,000 barrels of oil 1 year from today and 2 years from today.
  - The forward prices for deliver in 1 year and 2 years are \$110 and \$111/barrel.
  - The 1- and 2-year zero-coupon bond yields are 6% and 6.5%

Commodity swaps example (cont'd)

- Company can guarantee the cost of buying oil for the next 2 years by entering into long forward contracts for 100,000 barrels in each of the next 2 years.
- The PV of this cost per barrel is

$$110e^{-0.06} + 111e^{-0.065\times 2} = 201.063$$

- Typically a swap will call for equal payments each year.
- For example, the payment per barrel, x, should be such that

$$xe^{-0.06} + xe^{-0.065\times 2} = 201.063$$

- Then the no-arbitrage 2-year swap price per barrel is \$110.483

Commodity swaps example (cont'd)

- This example illustrates that swaps are equivalent to forward contracts coupled with borrowing and lending money.
- Consider the swap price of \$110.483/barrel versus the forward prices.
  - Relative to the forward curve price of \$110 in 1 year and \$111 in 2 years, we are overpaying by \$0.483 in the first year, and we are underpaying by \$0.517 in the second year.
- Thus, by entering into the swap, we are lending the counterparty money for 1 year. The implied interest rate on this loan is

$$e^r(0.483) = 0.517 \rightarrow r = 7\%$$

- Given the 1 and 2-year zero-coupon bond yields of 6% and 6.5%, 7% is the implied forward yield between years 1 and 2. (rounding error)
- The deal, which is fairly priced, has an embedded borrowing and lending rates equal to the implied forward rates in the yield curve.

Total Return Swaps (TRS)

- An exchange of an interest payment for the total return on a reference asset, paid periodically over the life of the TRS contract.
- Total Return = Cash Flows + (Change in Market Value)
- Fixed maturity date: Need not match reference asset maturity
- Reference Asset:
  - Bond, Loan (e.g., emerging market, sovereign, bank debt, mortgage-backed securities, or corporate loan), Index, Equity, Commodity
  - Payor need not own reference asset, but if it is owned, it hedges the cash flows for the payor.

TRS Swap - Example

- Total rate of return swap with notional principal \$10 million. Sold at LIBOR.
- Reference asset earns -10% over the period (interest and capital gain/loss)
- LIBOR is 4.5% over the period.
- What is the net cash flow on the swap on the payment date?
- Payor: pays $10\% + 4.5\%$

TRS Swap – Pricing Basics

- From the TROR payor's perspective
  - On each leg the swap is equivalent to a short position in the reference asset and a long position in a floating rate bond.
- From the TROR receiver's perspective
  - The swap is equivalent to a long position in the reference asset and a short position in a floating rate bond.
- The net swap value is zero at payment dates, after the total net return is exchanged.
  - Effectively, the swap restarts at each payment date on a fixed notional amount of assets
  - Between payment dates the value is the difference between the value of the floating rate bond and the reference asset.

Some motivations of TRS Receivers

- May gain off-balance sheet exposure to a desired asset class
- Reduce administrative costs (relative to outright purchase of reference asset)
  - Especially if underlying is illiquid or regulation prohibits direct ownership
- Provides a highly leveraged position, since no cash payments are initially made, and only net return is exchanged
  - Leverage is the main reason hedge funds tend to be TRS receivers.
  - But generally requires some collateral and/or capital, which reduces leverage.

Example: Creating leverage with TRS swap

Exposure to \$100 m of the reference asset:

	HF A	HF B	Cash investor
Asset yield	8.30%	8.30%	8.30%
Libor	-5.80%	-5.80%	
Net asset spread	2.50%	2.50%	
Spread to LIBOR	-1.00%	-1.00%	
Net swap spread	1.50%	1.50%	
Collateral	5%	10%	
Leverage	20 to 1	10 to 1	1 to 1
Interest on collateral	5.80%	5.80%	
Net return	35.80%	20.80%	8.30%

$$35.80\% = [.083(100) - .068(100) + .058(5)]/5$$

NB: the spread over LIBOR represents the default risk of the HFs.

Some Motivations of TROR Payers

- When owning the reference asset, you can hedge price risk and default risk of reference asset
- Avoid sales (tax consequences) and speculate on temporary price decline of asset
- Avoid restrictions on shorting

Duration for forwards, futures and swaps

Duration for forwards, futures and swaps

- Interest rate forwards, futures and swaps are often used in place of cash market delta and gamma hedging strategies
  - Advantages may include lower transactions costs and more liquidity
- The logic of delta and gamma hedging is the same using these derivatives
  - The goal is to take a position whose gains or losses will offset the gains or losses on the position being hedged.
- To implement an interest rate risk-management strategy with interest rate derivatives, we need to know how to calculate their duration and convexity.

Dollar duration of a forward contract on bonds

- A bond forward contract is equivalent to a portfolio consisting of a long and a short position in zero coupon bonds of equal value.
- For example, a long forward contract in a one-year bond to be delivered 3 years in the future:
  - Equivalent to a long position in a 4-year zero coupon bond, and a short position of equal value in a 3-year zero coupon bond.
  - Using this fact, we can derive an expression for the yield-sensitivity of the present value of the forward contract.
  - Assume $P_0(3) = P_0(4) = \$100$
  - $V_0 = P_0(4) - P_0(3) = 0$, $D_m(4) = 4/(1 + y/k)$, and $D_m(3) = 3/(1 + y/k)$

$$\begin{aligned} dV_0/dy &= dP_0(4)/dy - dP_0(3)/dy = -D_m(4)P_0(4) + D_m(3)P_0(3) \\ &= -(D_m(4) - D_m(3))100 = -(4-3)100/(1 + y/k) \end{aligned}$$

Using futures in a duration-based hedge

- Futures are effective instrument for hedging: low transaction costs, low couterparty risk, fast execution.

- However, there are relatively few futures contracts available. Hence, the duration of the obligation being hedged typically differs from the duration of the futures contract.

- Adjust the number of futures contracts to equate the sensitivity to yield change. Recall:

$$dP \approx -DPdy$$

- Similarly, if F is the contract price for an interest rate futures contract, then

$$dF \approx -D_F F dy$$

- The optimal hedge ratio is $h = \frac{\sigma_{P,F}}{\sigma_F} = \frac{dP}{dF} = \frac{dP/dy}{dF/dy} = \frac{DP}{D_F P_F}$.

- Hence, the number of futures contract required is $N = h\frac{Q}{Q_F} = \frac{DPQ}{D_F F Q_F}$.

  (Q_F is the size of one futures contract.)

Using futures in a duration-based hedge (cont'd)

- Example: On August 2, you have \$10 million bond portfolio, worrying about interest rate rising over the next 3 months.
- You decided to use the December T-bond futures contract to hedge the portfolio value.
  - Future price: $93'02 = 93 + 2/32 = 93.0625$ per \$100
  - Each contract is for the delivery of \$100,000 face value of bonds, hence $\$93,062.50$
- The duration of the bond portfolio is 6.80 years in December.
- The duration of the underlying (CTD) T-bond is 9.20 years at maturity of the futures.
- You "short" futures:

$$N = \frac{6.80}{9.20} \times \frac{\$10,000,000}{\$93,062.50} = 79.42$$

Swap duration

- Advantages of swaps: a wide variety of durations/convexities, higher liquidity at longer durations.
- Recall that for a fixed rate receiver, a swap is like having a portfolio that is long a fixed rate bond and short a floating rate bond.
- The effective duration of a (pure) floating rate bond is the time until the next reset, divided by $(1 + y/k)$
    - This is because the price of a floating rate bond between reset dates varies with short-term interest rates, but the price at the next reset date is fixed at par.
- The modified (and also effective) duration of the fixed rate bond can be calculated in the usual way.
- It follows that for the fixed rate receiver, the dollar duration of a swap, $-dP/dy = +P(\textit{fixed}) \times D(\textit{fixed}) - P(\textit{floating}) \times D_{\textit{eff}}(\textit{floating})$

Example

- Consider a new 5-year interest rate swap, offering a fixed rate of 6% (s.a.), and a floating rate of 6-mo LIBOR, with notional principal of \$1m. Assume current 6-mo LIBOR is also 6%.
  - What is the dollar duration for the fixed rate receiver?
  - What is the dollar duration for the floating rate receiver?
- A 5-year fixed rate bond with a 6% (s.a.) coupon selling at par has a modified duration of 4.265 years. The effective duration of the floating rate side is $0.5/(1.03) = 0.485$ years. The difference is 3.78 years.
  - The dollar duration of the swap is $3.78(\$1m)$
- The floating rate receiver's position is the negative of the fixed rate receiver's position.
  - The dollar duration of the swap is $-3.78(\$1m)$

LIBOR and Overnight Rates (Optional)

LIBOR vs Overnight reference rates

- LIBOR is being phased out.
- This is tricky for swaps.
  - For example, a 20-year swap negotiated at the end of 2013 will still have 10 years to run as of 2023.
- If banks stop providing LIBOR estimates, it will be necessary for the market to agree on a way of estimating LIBOR from the new reference rates.

Differences between LIBOR and the overnight reference rates

- LIBOR rates are the borrowing rates estimated by banks in the interbank market for periods between one day and one year.
- Overnight rates such as SOFR and SONIA are based on actual transactions between banks.
- The overnight rates are converted to longer reference rates using what might be termed an "averaging process." Usually the averaging involves daily compounding, but occasionally a simple arithmetic average is used (as for CME's one-month SOFR futures).
- LIBOR rates for a period are known at the beginning of the period to which they apply, whereas the result of the averaging process for overnight rates is known only at the end of the period.
- LIBOR rates incorporate some credit risk, whereas rates based on overnight rates such as SOFR and SONIA are considered to be risk-free rates.

Overnight Index Swaps

- Swaps based on overnight rates are becoming more popular. These are referred to as overnight indexed swaps (OISs).
- Consider a hypothetical two-year OIS initiated on March 8, 2022, between Apple and Citigroup.
  - Apple agrees to pay to Citigroup interest at the rate of 3% per annum every three months on a notional principal of $100 million, and in return Citgroup agrees to pay Apple the three-month SOFR floating reference rate on the same notional principal.
  - Assume that rates are quoted with quarterly compounding. Ignore the impact of day count conventions and holiday conventions

Overnight Index Swaps

Table 7.1 Cash flows to Apple for one possible outcome of the OIS in Figure 7.1. The swap lasts two years and the notional principal is $100 million.

Date	*SOFR rate (%)*	*Floating cash flow received ($'000s)*	*Fixed cash flow paid ($'000s)*	*Net cash flow ($'000s)*
June 8, 2022	2.20	550	750	−200
Sept. 8, 2022	2.60	650	750	−100
Dec. 8, 2022	2.80	700	750	−50
Mar. 8, 2023	3.10	775	750	+25
June 8, 2023	3.30	825	750	+75
Sept. 8, 2023	3.40	850	750	+100
Dec. 8, 2023	3.60	900	750	+150
Mar. 8, 2024	3.80	950	750	+200

- The difference between LIBOR swap and OIS
  - The LIBOR rate for a period is known at the beginning of the period, whereas the overnight reference rate is not known until the end of the period.
  - The 2.20% floating rate applicable to the first exchange on June 8, 2002, if LIBOR, would be known at the beginning of the swap's life on March 8, 2022