Practice Problem Set: Solutions

BUSS386 Futures and Options

1 Value at Risk

Solution:
The standard deviation of the daily change per asset is $1,000. The portfolio variance is

$$1,000^2 + 1,000^2 + 2 \times 0.3 \times 1,000 \times 1,000 = 2,600,000.$$

The portfolio's daily standard deviation is $\sqrt{2,600,000} \approx \$1,612.45$.
Over 5 days: $1,612.45 \times \sqrt{5} \approx \$3,605.55$.
Since the 1% tail corresponds to 2.326 standard deviations, the 5-day 99% VaR is

$$2.326 \times 3,605.55 \approx \$8,388.$$

2 Value at Risk

Solution:
Using the duration model:

$$\text{Daily change SD} = 5.2 \times 6,000,000 \times 0.0009 = \$28,080.$$

For 20 days: $28,080 \times \sqrt{20}$.
The 90% quantile for a normal variable (1.282 SD) gives

$$\text{VaR} = 1.282 \times 28,080 \times \sqrt{20} \approx \$160,990.$$

Weaknesses: This method assumes only parallel shifts and linear price–yield relationships, which may not hold in practice.

3 Value at Risk

Solution:
Portfolio variance:

$$(0.018 \times 300,000)^2 + (0.012 \times 500,000)^2 + 2 \times 300,000 \times 500,000 \times 0.6 \times 0.018 \times 0.012,$$

which equals 104.04 (in thousands^2).
SD $= \sqrt{104.04} \approx 10.2$ (in \$1,000s).
1-day 97.5% VaR $= 10.2 \times 1.96 \approx \$19,990$.
For 10 days: $19,990 \times \sqrt{10} \approx \$63,220$.
Diversification benefit (difference between sum of individual VaRs and portfolio VaR) $\approx$ \$7,438.

4 Interest Rate Conversion

Solution:
(a) Continuous compounding:

$$4\ln\big(1 + \tfrac{0.14}{4}\big) \approx 13.76\% \text{ per annum.}$$

(b) Annual compounding:

$$\big(1 + \tfrac{0.14}{4}\big)^4 - 1 \approx 14.75\% \text{ per annum.}$$

5 Bond Pricing

Solution:
Discounting at 10.4% (semiannual):

$$\text{Price} = \frac{4}{1.052} + \frac{4}{1.052^2} + \frac{104}{1.052^3} \approx 96.74.$$

Then solve for the 18-month zero rate R from

$$\frac{4}{1.05} + \frac{4}{1.05^2} + \frac{104}{(1 + R/2)^3} = 96.74,$$

giving $R \approx 10.42\%$.

6 Par Yield

Solution:
Set PV of cash flows = 100 and solve for coupon c.
The par yield $\approx 7.07\%$.

7 Forward Rates

Solution:
From zero rates, forward rates (approx):
Q2: 8.4%, Q3: 8.8%, Q4: 8.8%, Q5: 9.0%, Q6: 9.2%.

8 Forward Rate Agreements

Solution:
FRA-implied forward rate:

$$\frac{0.06 \times 0.75 - 0.05 \times 0.50}{0.25} \approx 8\%,$$

which exceeds the FRA rate of 7% $\Rightarrow$ arbitrage opportunity.

9 Forward Rate Agreements

Solution:
Convert semiannual rates to continuous compounding rates. For example, for 6 months, $e^{r_c} = (1 + r_m/m)^m = (1 + 4\%/2)^2$. Solve it for $r_c = 2 \times \ln(1 + 4\%/2) = 3.96\%$.

Compute forward rate for 6-month period starting in 18 months: $e^{1.5r_{1.5}}e^{0.5f} = e^{2r_2} \rightarrow f = 5.67\%$.

The value FRA on \$1m = $\$1m \times 5.91\% - \$1m \times 5.67\% = \$1,250$, where 5.91% is the c.c. rate for 6% s.a. rate.

10 Bond Prices and Interest Rate

Solution:
Bootstrapping zero rates, then forward rates and par yields: results as in table.
Two-year bond with 7% coupon: price $\approx$ 102.13, yield $\approx$ 5.77%.

11 Duration

Solution:
Duration measures sensitivity of bond prices to yield changes.
Limitations: assumes small, parallel shifts and linearity.

12 Duration

Solution:
Five-year bond:

$$\text{(a) Price} = 8e^{-0.11} + 8e^{-0.22} + 8e^{-0.33} + 8e^{-0.44} + 108e^{-0.55} \approx 86.80.$$

(b) Duration $\approx$ 4.256 years.
(c) A 0.2% yield decrease increases price by

$$86.80 \times 4.256 \times 0.002 \approx 0.74,$$

raising it to $\approx$ 87.54.
(d) At 10.8% yield, recalculated price = 87.54, confirming duration estimate.

13 Duration of Portfolios

Solution:
(a) Weighted average duration of Portfolio A = 5.95, same as B.
(b) For 0.1% yield increase, both fall $\approx$ 0.59%.
(c) For 5% yield increase, A declines $\approx$ 23.82%, B $\approx$ 25.73%, showing A's lower convexity.