Introduction to Derivatives

BUSS386. Futures and Options

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Lecture Outline

- Overview of derivatives/markets
- Review: measures of return and risk
- Reading: Hull, Ch. 1.1–1.10 and 22.1–22.3

What are derivatives?

- A derivative is a financial security (i.e., instrument, contract, asset) whose value depends on other underlying **variables**.
- Example: A contract to buy 50,000 barrels of crude oil on September 16, 2017, for $50 per barrel.
- Example: An option contract that gives the holder the right, but not the obligation, to buy 100 shares of a company's stock at $100 per share within the next three months.

What are the underlying variables?

- Usually, the price of a traded assets (e.g, equities, bonds, currencies, commodities)
- or some properties of asset prices (e.g, volatility)
- or some events (e.g., default)
- or weather (e.g. temperature, rainfall), inflation ...

$\Rightarrow$ All variables should be measurable and observable.

Type of derivatives

- Contract derivatives
  - Futures, forwards, swaps, options, warrants, callable bonds (embedded) etc.
  - The contract binds two counterparties to make a transaction at a future date. All profits and losses come from cash flows between the counterparties: zero-sum game
- Securitized or structured products
  - Securitization creates new derivative securities that receive and allocate the cash flows from the underlying pool to different classes of investors with different risk tolerance.
  - Collaterized mortage obligations, asset-backed securities, etc.
- A contract derivative **transfers** risk from one of the counterparties to the other. A securitized derivative **redistributes** risk that is inherent in the underlying assets.

History of derivatives

- Farmers and merchants have used derivatives for thousands of years.
  - 2000 B.C. in trade between India and the Arab Gulf
  - 300 B.C. olive growers in ancient Greece
- In the 12th centry, European merchants used forward contract for the future delivery of their goods
- During Amsterdam's tulip mania in the 1630s, derivatives helps some merchants from price swings
- In the 17th century, Japan developed a forward market in rice.
- Modern forms:
  - The Chicago Board of Trade (CBOT) was established in 1848 to trade futures.
  - The Chicago Mercantile Exchange (CME) was founded in 1919. (CBOT and CME later merged to form the CME Group).
  - The Chicago Board Options Exchange (CBOE) introduced call options in 1973 and put options in 1977.
  - In Korea, forex derivatives began trading in 1968, and exchanges were established in 1996.

Where to trade derivatives

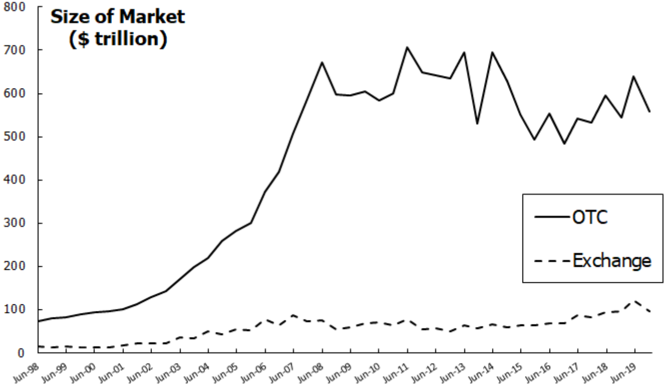
1. Exchange-traded market
    - Centralized Trading: All buy and sell orders are centralized in one place, either physically or electronically.
    - Standardized Contracts: Contracts are standardized, ensuring uniformity and reducing the risk of counterparty default.
    - Types of Derivatives: Futures and options are commonly traded.
    - Examples: National Stock Exchange of India $>$ B3 Brazil $>$ CME $>$ CBOE $>$ Intercontinental Exchange US $>$ NASDAQ $>$ Borsa Istanbul $>$ Zhengzhou Commodity Exchange $>$ Dalian Commodity Exchange $>$ Korea Exchange
    - Liquidity: High concentration of trades creates liquidity, which in turn attracts more liquidity.

Where to Trade Derivatives

2. Over-the-counter market
   - Decentralized Trading: There is no central place for collecting orders. Participants trade directly with each other or through a network of dealers.
   - Customizable Contracts: Contracts are not standardized and can be tailored to meet the specific needs of the participants.
   - Main Participants: Large institutions such as banks, hedge funds, and corporations.
   - Types of Derivatives: Forwards, swaps, options, and other customized derivatives are traded.

Where to Trade Derivatives

Market Types and Trading Volume



Source: Bank for International Settlement

What contributed to rapid growth?

"Necessity is the mother of invention" - Plato

- Deregulation, increased asset price volatility, and technological innovation
  - 1971: Currencies began to free float, leading to the introduction of currency futures in 1972.
  - 1973: The oil shock caused significant volatility in oil prices.
  - 1970s: Inflation and recessions resulted in volatile interest rates.
  - 1978: Deregulation of natural gas.
  - 1990s: Deregulation of electricity markets.

Why are derivatives useful?

- Derivatives facilitate the transfer of risk from those who are exposed to it to those more willing to bear it, making them a powerful tool for risk management.
- While risk management often aims to reduce risk, it can also involve strategically assuming risks that offer potential benefits.
- By effectively redistributing risk, derivatives enable productive activities that might otherwise be deemed too risky to pursue.
- However, derivatives can be misused, which is why regulations exist to mitigate potential abuses and ensure market stability.

Dangers of derivates trading

- Without proper risk management, derivatives trading can lead to significant losses. Here are some notable examples:
  - Societe Generale (2008): Jerome Kerviel lost over $7 billion by speculating on the future direction of equity indices.
  - UBS (2011): Kweku Adoboli lost $2.3 billion by taking unauthorized speculative positions in stock market indices.
  - Shell (1993): A single employee in the Japanese subsidiary of Shell lost $1 billion in unauthorized trading of currency futures.
  - Barings Bank (1995): Nick Leeson lost £827 million, leading to the bank's collapse.
  - Long-Term Capital Management (1998): The hedge fund lost $4.6 billion due to high-risk arbitrage trading strategies.
  - AIG (2008): AIG faced a liquidity crisis due to losses on credit default swaps, leading to a $182 billion government bailout.
- Risk management is a critically important task.
  - Define risk, set risk limit, perform various (created) scenario analysis.

The OTC Market Prior to 2008

- The OTC market was largely unregulated.
- Banks acted as market makers, quoting bid and ask prices.
- Transactions between two parties were usually governed by master agreements provided by the International Swaps and Derivatives Association (ISDA).[1]
- Some transactions were cleared through central counterparties (CCPs), which act as intermediaries between the two sides of a transaction, similar to an exchange.

[1] The ISDA is a trade organization of participants in the market for over-the-counter derivatives. ISDA has created a standardized contract (the ISDA Master Agreement to govern derivative transactions, which helps to reduce legal and credit risks.

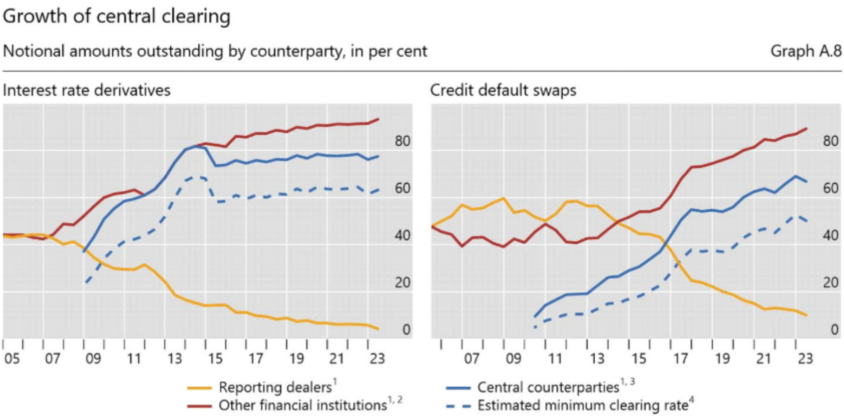
Since 2008...

- OTC market has become more regulated. Objectives:
  - Reduce systemic risk
  - Increase transparency
- In the U.S. and other countries, collateral and clearing of trades through a central clearing house (CCP) are required for all standard OTC contracts.
- CCPs must be used to clear standardized transactions between financial institutions in most countries.
- All trades must be reported to a central repository

The Lehman Bankruptcy

- Lehman Brothers filed for bankruptcy on September 15, 2008, marking the largest bankruptcy in U.S. history.
- Lehman was heavily involved in the OTC derivatives markets and faced financial difficulties due to high-risk activities and an inability to roll over its short-term funding.
- The firm had hundreds of thousands of outstanding transactions with approximately 8,000 counterparties.
- The process of unwinding these transactions has been challenging for both Lehman's liquidators and their counterparties.

Central Clearing



[1] As a percentage of notional amounts outstanding against all counterparties. [2] Including central counterparties but excluding reporting dealers. [3] For interest rate derivatives, data for CCPs prior to end-June 2016 are estimated by indexing the amounts reported at end-June 2016 to the growth since 2008 of notional amounts outstanding cleared through LCH's SwapClear service. [4] Proportion of trades that are cleared, estimated as (CCP / 2) / (1 – (CCP / 2)), where CCP represents the share of notional amounts outstanding that dealers report against CCPs. The CCP share is halved to adjust for the potential double-counting of inter-dealer trades novated to CCPs.

Sources: LCH.Clearnet Group Ltd; BIS OTC derivatives statistics (Table D7 and Table D10.1); BIS calculations.

Source: Bank for International Settlement

Who trade derivatives?

Derivatives are traded by various market participants:

- Corporations: Hedge future cash flows and manage risks (e.g., fuel futures for airlines).
- Financial Institutions: Manage risks and offer risk management solutions (e.g., interest rate swaps).
- Hedge Funds: Achieve higher returns through leverage and complex strategies.
- Market Makers (Dealers): Provide liquidity and profit from bid-ask spreads.
- Financial Engineers: Design new derivative products to meet specific needs.

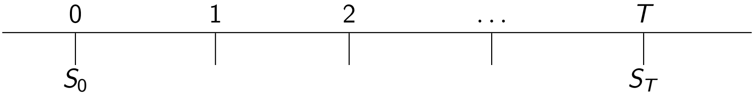
Each participant contributes to the market's depth and liquidity.

Statistics: Review

Investment and Risk

- When investing in financial assets, we are often uncertain about future value (risks).

- Stock investment



- At time 0, we are uncertain about the future return on stock, $\left(\frac{S_T}{S_0} - 1\right)$.

Investment and Risk

- What if we want to compare a risky investment to a risk-free investment?

e.g. You are presented with the two investment projects. Which one would you choose?

	Project 1	Project 2	
		good	bad
return	fixed 5%	10%	0%

- For decisions like this, we consider the probability of risky outcomes.

Random Variables - Discrete

- Suppose that a random return R can take one of the following values.

return	r_1	r_2	$\cdots$	r_n
probability	p_1	p_2	$\cdots$	p_n

- The expectation of the return is

$$E(R) = \sum_{i=1}^{n} r_i \times p_i$$

- The variance of the return is

$$\text{Var}(R) = \sum_{i=1}^{n} (r_i - E(R))^2 \times p_i$$

- The standard deviation is

$$\sigma(R) = \sqrt{\text{Var}(R)}$$

Random Variables - Continuous

- Suppose that the return R is a continuous random variable that can take any value from $(-\infty, \infty)$.

- The probability density function $f(r)$ is given.

- Using $f(r)$, we can calculate the probability of any event. For example, the probability that the return is lower than 0.05 is

$$\text{Prob}(R \leq 0.05) = \int_{-\infty}^{0.05} f(r)dr$$

- The expectation of the return is

$$E(R) = \int_{-\infty}^{\infty} r \times f(r)dr$$

Normal Random Variables

- Consider a random variable R with the following probability density function

$$f(R) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(R-\mu)^2}{2\sigma^2}}.$$

We call R normally distributed with mean μ and standard deviation σ. To simplify, we also express as follows:

$$R \sim N(\mu, \sigma^2)$$

- If we multiply R by a and add b, the result is also normally distributed

$$aR + b \sim N(a\mu + b, a^2\sigma^2)$$

Standard Normal Random Variables

- Consider a normal random variable R with $\mu = 0$ and $\sigma = 1$. In other words, $R \sim N(0, 1)$. We call it a standard normal random variable.

- Suppose that we want to find the probability that R is lower than x. Graphically, this probability is the shadowed area in the figure below:

Figure 14.3 Shaded area represents $N(x)$.

Standard Normal Random Variables

- To find this probability, we calculate

$$\text{Prob}(R \leq x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{r^2}{2}} dr \equiv \Phi(x).$$

  $\Phi(x)$ is called the cumulative probability distribution function for a standard normal random variable.

- For any x, the value of $\Phi(x)$ can be found using the excel function, *norm.s.dist*(x, TRUE).

Standard Normal Random Variables

Ex.1 Suppose that $R_1 \sim \phi(0,1)$. What is the probability that R_1 is larger than 1?

Ex.2 Suppose that $R_2 \sim \phi(0.1, 0.2)$. What is the probability that R_2 is equal to or smaller than 0.5?

Risk Measures

Risk Measures

- Companies need to assess and manage risks to prevent business failures.
- To have a sense of how risky a project or business is, we can refer to the probability distribution of possible outcomes.
- There are multiple risk measures
  - Standard Deviation
  - Value at Risk (VaR)
  - Expected Shortfalls
  - ...
- Different measures focus on different aspects of the distribution.

Standard Deviation

- Standard deviation measures the level of uncertainty about the outcomes, or the dispersion of probability distribution.

- The larger standard deviation is, the riskier a project.

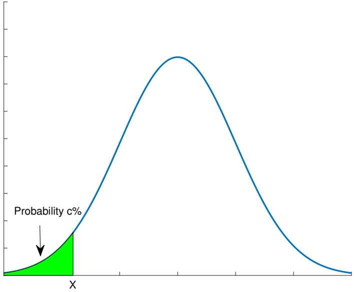
Ex. Consider the following two projects. Which is riskier?

	Project 1		Project 2	
	good	bad	good	bad
return	10%	0%	0%	-10%
probability	0.5	0.5	0.5	0.5

- A disadvantage of the standard deviation is that it cannot distinguish between upside and downside movement.

Value at Risk

- Value at Risk (VaR) represents the potential loss in value of a portfolio, given a certain probability over a specific time period.
  - E.g. With a 5% probability, our portfolio may experience a loss greater than the VaR amount over the next one month. I.e., There is a 95% probability that our loss will not excees the VaR amount.



That is, we want to find X such that

$$\text{Prob}\,(R \leq X) = 0.05$$

Value at Risk

- How can we find X satisfying $\Pr(R \leq X) = 0.05$, i.e., 95% VaR?
- In a special case when $R \sim \phi(\mu, \sigma)$, we can find X using the Excel function `norm.inv()`.[2]
  - For given $1 - p$, `norm.inv(1-p,`μ, σ`)` is X that satisfies $\text{Prob}(R \leq X) = 1 - p$.

VaR at 5% = `norm.inv`(0.05,0,1) = -1.645

VaR at 10% = `norm.inv`(0.1,0,1) = -1.282

[2] Closed-form: $VaR(X) = \Phi^{-1}(1 - p)\sigma + \mu$

Value at Risk - Example

Q. Suppose that we own a stock whose return is normally distributed with the mean 15% and the standard deviation 30%. What is a 5% loss on this stock?

Answer: Let X denote the 5% loss. Then,
$\Pr(R \leq X) = \texttt{norm.inv}(0.05, 0.15, 0.30) = -34.3\%$

Value at Risk - Example

Q. A portfolio worth \$10 million has a 1-day standard deviation of \$200,000 and an approximate mean of zero. Assume that the change is normally distributed. What is the 1-day 99% VaR for our portfolio consisting of a \$10 million position? What is the 10-day 99% VaR?

Answer: `norm.s.inv(0.01)` $= -2.326$, meaning that there is a 1% probability that a normally distributed variable will decrease in value by more than 2.326 standard deviations.

Hence, 1-day 99% VaR is $2.326 \times \$200,000 = \$465,300$.

The 10-day 99% VaR is $2.326 \times (\$200,000 \times \sqrt{N}) = \$1,471,300$.

Value at Risk - Multiple Stocks

- Consider a portfolio consisting of n different stocks.
- The return on the portfolio is

$$R_p = \sum_{i=1}^{n} w_i R_i$$

where w_i is the fraction of wealth invested in stock i.

- If each stock return is normally distributed, then the portfolio return is also normally distributed.

Value at Risk - Example

Q. Consider a portfolio consisting of stock A and stock B. In the portfolio, \$5 million are invested in each of stock A and stock B. The return on each stock is normally distributed. Stock A has an expected return of 15% and a standard deviation of 30%. Stock B has an expected return of 18% and a standard deviation of 45%. The correlation between stock A and stock B is 0.4. What is the 90% VaR for the portfolio?

NB When $X \sim \phi(\mu_x, \sigma_x^2)$ and $Y \sim \phi(\mu_y, \sigma_y^2)$, then $X + Y \sim \phi(\mu_x + \mu_y, \sigma_x^2 + \sigma_y^2 + 2\rho\sigma_x\sigma_y)$

Answer: The expected return of the portfolio is:

$$\mu_p = 0.5 \times 0.15 + 0.5 \times 0.18 = 0.165 \text{ or } 16.5\%$$

- The standard deviation of the portfolio is:

$$\sigma_p = \sqrt{(0.5 \times 0.30)^2 + (0.5 \times 0.45)^2 + 2(0.5)(0.5)(0.4)(0.30)(0.45)} = 0.315$$

- The 90% VaR for the portfolio is:

$$\text{VaR}_{90\%} = \mu_p + \sigma_p \times \texttt{norm.s.inv}(0.10) = 0.165 + 0.315 \times (-1.282) = -0.239$$

- Therefore, the 90% VaR for the \$10 million portfolio is:

$$10,000,000 \times 0.239 = \$2,390,000$$

Value at Risk - Historical Data

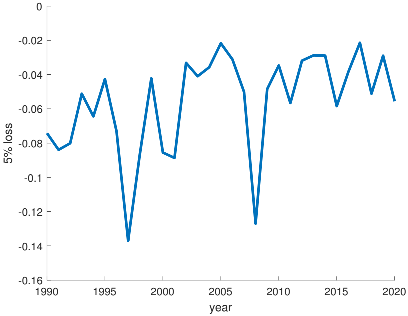
- We can also calculate the VaR using historical data without assuming a specific distribution.
- For example, let's consider 1-year-long historical data of daily returns for a stock price index.
- We aim to estimate the 5% VaR for the next day's return.
- To do this, we assume that the next day's return will be similar to one of the past year's returns.
- The 5% VaR is then the 5th percentile of these historical returns.

Value at Risk - Some Issues I

- VaR estimation is based on the assumption that the distribution of future return is the same as (at least similar to) the distribution of past return.

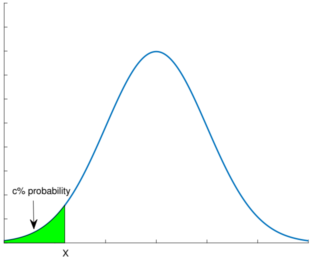
- This assumption may not hold in the real world.

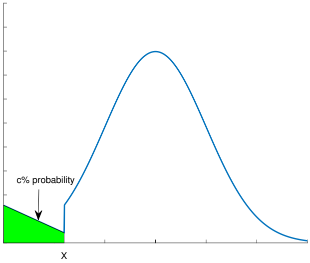
VaR for Index (lowest 5% daily returns)



Value at Risk - Some Issues II

- VaR specifies the **minimum** loss that will occur with a given probability.
- VaR tells nothing about the expected magnitude of the loss.
- Which is the better between the following two?





Expected Shortfall

- Expected Shortfall is another measure to address the shortcoming of VaR.
    - It asks "If things get bad, what is the expected loss?"

- Suppose that we focus on the loss that will happen with 5% probability. Let V denote the 5% loss (VaR). Then, [3]

$$\text{Expected shortfall} = E\left(R \mid R \leq V\right)$$

[3] Under normal distribution: Expected shortfall $= \mu - \sigma \frac{\phi((V-\mu)/\sigma)}{\Phi((V-\mu)/\sigma)}$

Expected Shortfall

- Once historical data are given, we can compute the expected shortfall.
  - In Excel, use "averageif()".

Ex. Let's use the 1-year-long data of daily returns on a stock index.

Q1. What is the expected shortfall with 5% probability?

Q2. What is the expected shortfall with 10% probability?

Application: Bank Regulation

- VaR and ES are widely used in the financial industry to measure and manage risk.
- The Basel Committee on Banking Supervision (BCBS) provides global banking regulations.
  - 1996 Amendment: Required capital $= k \times VaR(1\%, 10 days)$, where $k \geq 3$.
  - Basel II (2007): Suggested VaR(0.1%, 1-year) for risk assessment.
  - Basel IV (2021): Recommended 97.5% expected shortfall (ES) for a comprehensive risk view.